Exercise 1.1

Question 1:

Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Answer:

(i) 135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

$225 = 135 \times 1 + 90$

Since remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to obtain

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

(ii)196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii)867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

$867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$255 = 102 \times 2 + 51$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$102 = 51 \times 2 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

Question 2:

Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Answer:

Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm, $a = 6q + r$for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2\,(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2\,(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$,

or $6q + 3$,

or $6q + 5$

Question 3:

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer:

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

Question 4:

Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m.

[**Hint:** Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Answer:

Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$a^2 = (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2$$

$$a^2 = (9q^2) \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4$$

$$= 3 \times (3q^2) \text{ or } 3(3q^2 + 2q) + 1 \text{ or } 3(3q^2 + 4q + 1) + 1$$

$$= 3k_1 \text{ or } 3k_2 + 1 \text{ or } 3k_3 + 1$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Question 5:

Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Answer:

Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$\therefore a = 3q$ or $3q+1$ or $3q+2$

Therefore, every number can be represented as these three forms.

There are three cases.

Case 1: When $a = 3q$,

$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m,$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$a^3 = (3q+1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$a^3 = (3q+2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Exercise 1.2

Question 1:

Express each number as product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Answer:

(i) $140 = 2\times2\times5\times7 = 2^2\times5\times7$

(ii) $156 = 2\times2\times3\times13 = 2^2\times3\times13$

(iii) $3825 = 3\times3\times5\times5\times17 = 3^2\times5^2\times17$

(iv) $5005 = 5\times7\times11\times13$

(v) $7429 = 17\times19\times23$

Question 2:

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Answer:

(i) 26 and 91

$26 = 2\times13$

$91 = 7\times13$

$\text{HCF} = 13$

$\text{LCM} = 2\times7\times13 = 182$

Product of the two numbers $= 26\times91 = 2366$

$\text{HCF}\times\text{LCM} = 13\times182 = 2366$

Hence, product of two numbers = HCF × LCM

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$HCF = 2$

$LCM = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of the two numbers $= 510 \times 92 = 46920$

$HCF \times LCM = 2 \times 23460$

$= 46920$

Hence, product of two numbers = HCF × LCM

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$54 = 2 \times 3^3$

$HCF = 2 \times 3 = 6$

$LCM = 2^4 \times 3^3 \times 7 = 3024$

Product of the two numbers $= 336 \times 54 = 18144$

$HCF \times LCM = 6 \times 3024 = 18144$

Hence, product of two numbers = HCF × LCM

Question 3:

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Answer:

(i) 12,15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$\text{HCF} = 3$

$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17,23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

$\text{HCF} = 1$

$\text{LCM} = 17 \times 23 \times 29 = 11339$

(iii) 8,9 and 25

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

$\text{HCF} = 1$

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

Question 4:

Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer:

$\text{HCF}(306, 657) = 9$

We know that, $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$\text{LCM} = 22338$

Question 5:

Check whether 6^n can end with the digit 0 for any natural number n.

Answer:

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as 10 = 2 × 5

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Question 6:

Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Answer:

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Question 7:

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer:

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$18 = 2 \times 3 \times 3$

And, $12 = 2 \times 2 \times 3$

LCM of 12 and 18 = $2 \times 2 \times 3 \times 3 = 36$

Therefore, Ravi and Sonia will meet together at the starting pointafter 36 minutes.

Exercise 1.3

Question 1:

Prove that $\sqrt{5}$ is irrational.

Answer:

Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Let a and b have a common factor other than 1. Then we can divide them by the common factor, and assume that a and b are co-prime.

$$a = \sqrt{5}b$$
$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

Let $a = 5k$, where k is an integer

$$(5k)^2 = 5b^2$$

$b^2 = 5k^2$ This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Question 2:

Prove that $3+2\sqrt{5}$ is irrational.

Answer:

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$$3+2\sqrt{5}=\frac{a}{b}$$

$$2\sqrt{5}=\frac{a}{b}-3$$

$$\sqrt{5}=\frac{1}{2}\left(\frac{a}{b}-3\right)$$

Since *a* and *b* are integers, $\frac{1}{2}\left(\frac{a}{b}-3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false. Therefore, $3+2\sqrt{5}$ is irrational.

Question 3:

Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6+\sqrt{2}$

Answer:

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$\frac{1}{\sqrt{2}} = \frac{a}{b}$

$\sqrt{2} = \frac{b}{a}$

$\frac{b}{a}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$7\sqrt{5} = \frac{a}{b}$ for some integers *a* and *b*

$\therefore \sqrt{5} = \frac{a}{7b}$

$\frac{a}{7b}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6+\sqrt{2}$

Let $6+\sqrt{2}$ be rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$$6+\sqrt{2}=\frac{a}{b}$$

$$\sqrt{2}=\frac{a}{b}-6$$

Since *a* and *b* are integers, $\frac{a}{b}-6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is false and hence, $6+\sqrt{2}$ is irrational.

Exercise 1.4

Question 1:

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Answer:

(i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455 = 5 \times 7 \times 13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$1600 = 2^6 \times 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$343 = 7^3$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 \times 5^2}$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15} = \frac{2\times3}{3\times5} = \frac{2}{5}$

The denominator is of the form 5^n.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50} = \frac{7\times5}{10\times5} = \frac{7}{10}$

$10 = 2\times5$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210} = \frac{11\times7}{30\times7} = \frac{11}{30}$

$30 = 2\times3\times5$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 3 as its factors, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

Question 2:

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer:

(i) $\frac{13}{3125} = 0.00416$

$$\begin{array}{r|l} 3125 & 13.00000 \\ \end{array}$$

Quotient: 0.00416

$$\begin{array}{l} 0 \\ \hline 130 \\ 0 \\ \hline 1300 \\ 0 \\ \hline 13000 \\ 12500 \\ \hline 5000 \\ 3125 \\ \hline 18750 \\ 18750 \\ \hline \times \\ \hline \end{array}$$

(ii) $\frac{17}{8} = 2.125$

$$\begin{array}{r|l} 8 & 17 \\ \end{array}$$

Quotient: 2.125

$$\begin{array}{l} 16 \\ \hline 10 \\ 8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline \times \\ \hline \end{array}$$

(iv) $\frac{15}{1600} = 0.009375$

$$
\begin{array}{r}
0.009375 \\
1600\overline{)15.000000} \\
0 \\
\hline
150 \\
0 \\
\hline
1500 \\
0 \\
\hline
15000 \\
14400 \\
\hline
6000 \\
4800 \\
\hline
12000 \\
11200 \\
\hline
8000 \\
8000 \\
\hline
\times \\
\hline
\end{array}
$$

(vi) $\frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$

$$
\begin{array}{r}
0.115 \\
200\overline{)23.000} \\
0 \\
\hline
230 \\
200 \\
\hline
300 \\
200 \\
\hline
1000 \\
1000 \\
\hline
\times \\
\hline
\end{array}
$$

(viii) $\frac{6}{15}=\frac{2\times3}{3\times5}=\frac{2}{5}=0.4$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ 0 \\ \hline 20 \\ 20 \\ \hline \times \\ \hline \end{array}$$

(ix) $\frac{35}{50}=0.7$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ 0 \\ \hline 350 \\ 350 \\ \hline \times \\ \hline \end{array}$$

Question 3:

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of q?

(i) 43.123456789 (ii) 0.120120012000120000… (iii) $43.\overline{123456789}$

Answer:

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.12012001200012000

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

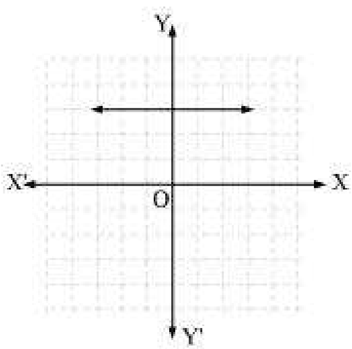
Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.

Exercise 2.1

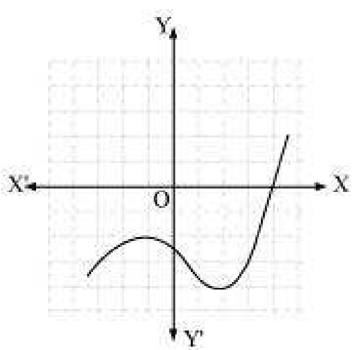
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

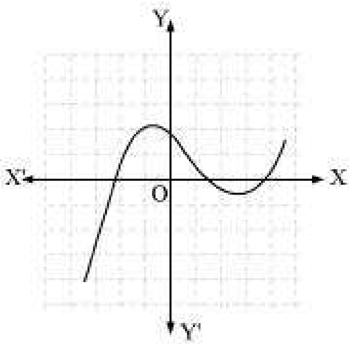
(i)



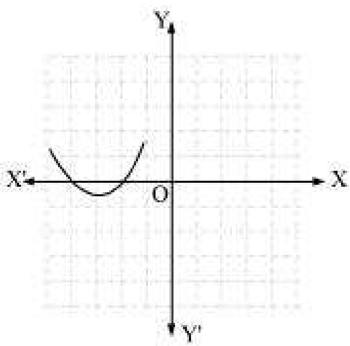
(ii)



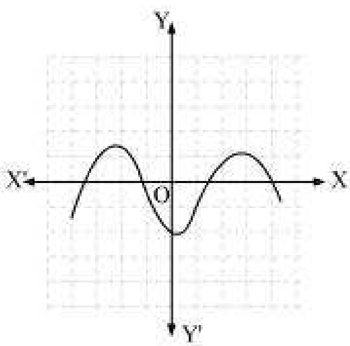
(iii)



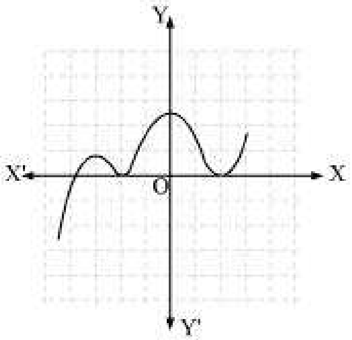
(iv)



(v)



(v)



Answer:

(i) The number of zeroes is 0 as the graph does not cut the *x*-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the *x*-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the *x*-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the *x*-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(\text{i})\, x^2-2x-8$ $(\text{ii})\, 4s^2-4s+1$ $(\text{iii})\, 6x^2-3-7x$

$(\text{iv})\, 4u^2+8u$ $(\text{v})\, t^2-15$ $(\text{vi})\, 3x^2-x-4$

Answer:

$(\text{i})\qquad x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when $x - 4 = 0$ or $x + 2 = 0$, i.e., when $x = 4$ or $x = -2$

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$(\text{ii})\qquad 4s^2-4s+1=(2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e.,

$x=\frac{-1}{3}$ or $x=\frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3}\times\frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2+8u=4u^2+8u+0$

$=4u(u+2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and −2.

Sum of zeroes = $0+(-2)=-2=\frac{-(8)}{4}=\frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v) t^2-15

$=t^2-0t-15$

$=(t-\sqrt{15})(t+\sqrt{15})$

The value of $t^2 - 15$ is zero when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., when $t=\sqrt{15}$ or $t=-\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15}+\left(-\sqrt{15}\right)=0=\frac{-0}{1}=\frac{-(\text{Coefficient of } t)}{\left(\text{Coefficient of } t^2\right)}$

Product of zeroes = $\left(\sqrt{15}\right)\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(vi) $3x^2-x-4$

$=(3x-4)(x+1)$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e., when $x=\frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1.

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) $0,\sqrt{5}$

(iv) $1,1$ (v) $-\frac{1}{4},\frac{1}{4}$ (vi) $4,1$

Answer:

(i) $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

If $a = 1$, then $b = 0$, $c = \sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1, 1

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
If $a=1$, then $b=-1$, $c=1$

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4},\frac{1}{4}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{-1}{4}=\frac{-b}{a}$$
$$\alpha\times\beta=\frac{1}{4}=\frac{c}{a}$$
If $a=4$, then $b=1$, $c=1$

Therefore, the quadratic polynomial is $4x^2+x+1$.

(vi) 4, 1

Let the polynomial be ax^2+bx+c.

$$\alpha+\beta=4=\frac{4}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
If $a=1$, then $b=-4$, $c=1$

Therefore, the quadratic polynomial is x^2-4x+1.

Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x)=x^3-3x^2+5x-3, \qquad g(x)=x^2-2$

(ii) $p(x)=x^4-3x^2+4x+5, \qquad g(x)=x^2+1-x$

(iii) $p(x)=x^4-5x+6, \qquad g(x)=2-x^2$

Answer:

(i) $p(x)=x^3-3x^2+5x-3$

$q(x)=x^2-2$

$$
\begin{array}{r|l}
 & x-3 \\
\hline
x^2-2 & x^3-3x^2+5x-3 \\
 & x^3 \qquad\quad -2x \\
 & - \qquad\quad + \\
\hline
 & -3x^2+7x-3 \\
 & -3x^2 \qquad +6 \\
 & + \qquad\qquad - \\
\hline
 & 7x-9 \\
\hline
\end{array}
$$

Quotient = $x-3$

Remainder = $7x-9$

(ii) $p(x)=x^4-3x^2+4x+5=x^4+0.x^3-3x^2+4x+5$

$q(x)=x^2+1-x=x^2-x+1$

$$
\begin{array}{r|l}
 & x^2+x-3 \\
\hline
x^2-x+1 & x^4+0.x^3-3x^2+4x+5 \\
 & x^4-\ \ x^3+x^2 \\
 & -\ \ +\ \ \ - \\
\hline
 & \qquad x^3-4x^2+4x+5 \\
 & \qquad x^3-\ x^2+\ x \\
 & \qquad -\ \ \ +\ \ \ - \\
\hline
 & \qquad\quad -3x^2+3x+5 \\
 & \qquad\quad -3x^2+3x-3 \\
 & \qquad\quad +\ \ \ -\ \ \ + \\
\hline
 & \qquad\qquad\qquad 8 \\
\hline
\end{array}
$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2-2 \\
\hline
-x^2+2 & x^4+0.x^2-5x+6 \\
 & x^4-2x^2 \\
 & -\ \ + \\
\hline
 & \qquad 2x^2-5x+6 \\
 & \qquad 2x^2\qquad -4 \\
 & \qquad -\qquad\ \ + \\
\hline
 & \qquad\quad -5x+10 \\
\hline
\end{array}
$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2-3, 2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1, 3x^4+5x^3-7x^2+2x+2$

(iii) $x^3-3x+1, x^5-4x^3+x^2+3x+1$

Answer:

(i) $t^2-3,\ 2t^4+3t^3-2t^2-9t-12$

$t^2-3 = t^2+0.t-3$

$$
\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^2+0.t-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4+0.t^3-6t^2 \\
 & -\quad -\quad + \\
\hline
 & 3t^3+4t^2-9t-12 \\
 & 3t^3+0.t^2-9t \\
 & -\quad -\quad + \\
\hline
 & 4t^2+0.t-12 \\
 & 4t^2+0.t-12 \\
 & -\quad -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\ \hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & 3x^4+9x^3+3x^2 \\
 & -\quad -\quad - \\ \hline
 & -4x^3-10x^2+2x+2 \\
 & -4x^3-12x^2-4x \\
 & +\quad +\quad + \\ \hline
 & 2x^2+6x+2 \\
 & 2x^2+6x+2 \\ \hline
 & 0 \\ \hline
\end{array}
$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

$$
\begin{array}{r|l}
 & x^2-1 \\ \hline
x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\
 & x^5-3x^3+x^2 \\
 & -\quad +\quad - \\ \hline
 & -x^3 \qquad +3x+1 \\
 & -x^3 \qquad +3x-1 \\
 & + \qquad\quad -\quad + \\ \hline
 & 2 \\ \hline
\end{array}
$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Question 3:

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer:

$p(x)=3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
\hline
x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\
\hline
 & \qquad 6x^3+3x^2-10x-5 \\
 & \qquad 6x^3+0x^2-10x \\
 & \qquad -\quad -\quad + \\
\hline
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad -\quad -\quad + \\
\hline
 & \qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

$$3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$$

$$=3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$

$x = -1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, −1 and −1.

Question 4:

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Answer:

$p(x) = x^3 - 3x^2 + x + 2 \qquad (\text{Dividend})$

$g(x)$ = ? (Divisor)

Quotient = (x − 2)

Remainder = (− 2x + 4)

Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$$
$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x-2)$$
$$x^3 - 3x^2 + 3x - 2 = g(x)(x-2)$$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x-2)$

$$\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x-2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & \quad -x^2 + 3x - 2 \\
 & \quad -x^2 + 2x \\
 & \quad + \quad - \\
\hline
 & \qquad\quad x - 2 \\
 & \qquad\quad x - 2 \\
 & \qquad\quad - \quad + \\
\hline
 & \qquad\qquad 0
\end{array}$$

$\therefore g(x) = (x^2 - x + 1)$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x)$ = 0

Answer:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with

$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2+2x+2$ by 2.

Here, $p(x) = 6x^2+2x+2$

$g(x) = 2$

$q(x) = 3x^2+x+1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2+2x+2 = 2(3x^2+x+1)$

$= 6x^2+2x+2$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Exercise 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$

Answer:

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$
$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$
$$= 0$$

$$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$$
$$= 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$
$$= -16 + 4 + 10 + 2 = 0$$

Therefore, $\frac{1}{2}$, 1, and −2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain a = 2, b = 1, c = −5, d = 2

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$
$$= 8 - 16 + 10 - 2 = 0$$

$$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$$
$$= 1 - 4 + 5 - 2 = 0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain *a* = 1, *b* = −4, *c* = 5, *d* = −2.

Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1)

=2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $= \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Answer:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta, \text{ and } \gamma$.

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If *a* = 1, then *b* = −2, *c* = −7, *d* = 14

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find *a* and *b*.

Answer:

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are *a* − *b*, *a* + *a* + *b*

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$p = 1$, $q = -3$, $r = 1$, $t = 1$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are $1-b, 1, 1+b$.

$$\text{Multiplication of zeroes} = 1(1-b)(1+b)$$

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Question 4:

]It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer:

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4-6x^3-26x^2+138x-35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+\quad x^2 \\
 & -\quad +\quad - \\
\hline
 & -2x^3-27x^2+138x-35 \\
 & -2x^3+8x^2-\quad 2x \\
 & +\quad -\quad + \\
\hline
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & +\quad -\quad + \\
\hline
 & 0
\end{array}
$$

Clearly, $x^4-6x^3-26x^2+138x-35 = \left(x^2-4x+1\right)\left(x^2-2x-35\right)$

It can be observed that $\left(x^2-2x-35\right)$ is also a factor of the given polynomial.

And $\left(x^2-2x-35\right) = (x-7)(x+5)$

Therefore, the value of the polynomial is also zero when $x-7=0$ or $x+5=0$

Or $x = 7$ or -5

Hence, 7 and −5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be *x* + *a*, find *k* and *a*.

Answer:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

$x^4-6x^3+16x^2-25x+10-x-a=x^4-6x^3+16x^2-26x+10-a$ will be perfectly divisible by x^2-2x+k.

Let us divide $x^4-6x^3+16x^2-26x+10-a$ by x^2-2x+k

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-26x+10-a \\
 & x^4-2x^3+kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3+(16-k)x^2-26x \\
 & -4x^3+8x^2-4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2-(26-4k)x+10-a \\
 & (8-k)x^2-(16-2k)x+(8k-k^2) \\
 & - \quad + \quad - \\
\hline
 & (-10+2k)x+(10-a-8k+k^2)
\end{array}
$$

It can be observed that $(-10+2k)x+(10-a-8k+k^2)$ will be 0.

Therefore, $(-10+2k)$ = 0 and $(10-a-8k+k^2)$ = 0

For $(-10+2k)$ = 0,

2 k =10

And thus, k = 5

For $(10-a-8k+k^2)$ = 0

10 − a − 8 × 5 + 25 = 0

10 − a − 40 + 25 = 0

− 5 − a = 0

Therefore, a = −5

Hence, k = 5 and a = −5

Exercise 3.1

Question 1:

Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Answer:

Let the present age of Aftab be x.

And, present age of his daughter = y

Seven years ago,

Age of Aftab = $x - 7$

Age of his daughter = $y - 7$

According to the question,

$(x-7) = 7(y-7)$

$x - 7 = 7y - 49$

$x - 7y = -42 \quad (1)$

Three years hence,

Age of Aftab = $x + 3$

Age of his daughter = $y + 3$

According to the question,

$(x+3) = 3(y+3)$

$x + 3 = 3y + 9$

$x - 3y = 6 \quad (2)$

Therefore, the algebraic representation is

$x - 7y = -42$

$x - 3y = 6$

For $x - 7y = -42$,

$x = -42 + 7y$

The solution table is

x	− 7	0	7
y	5	6	7

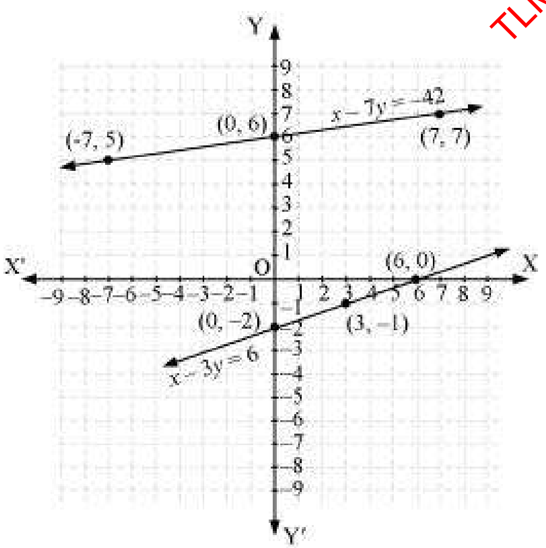
For $x-3y=6$,

$x=6+3y$

The solution table is

x	6	3	0
y	0	− 1	− 2

The graphical representation is as follows.



Question 2:

The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 2 more balls of the same kind for Rs 1300. Represent this situation algebraically and geometrically.

Answer:

Let the cost of a bat be Rs *x*.

And, cost of a ball = Rs *y*.

According to the question, the algebraic representation is

$$3x + 6y = 3900$$
$$x + 2y = 1300$$

For $3x + 6y = 3900$,

$$x = \frac{3900 - 6y}{3}$$

The solution table is

x	300	100	− 100
y	500	600	700

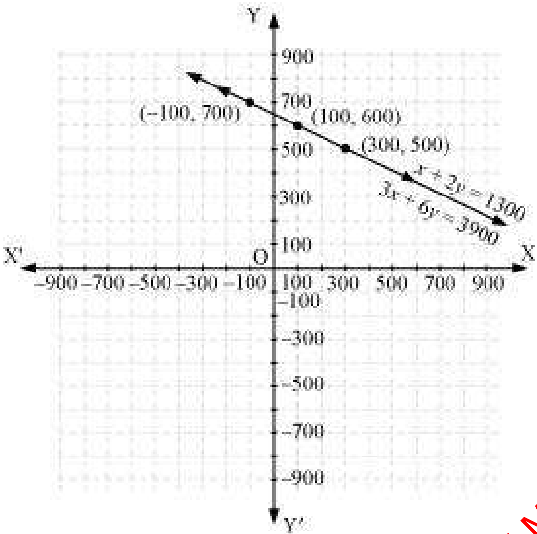
For $x + 2y = 1300$,

$$x = 1300 - 2y$$

The solution table is

x	300	100	− 100
y	500	600	700

The graphical representation is as follows.



Question 3:

The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Answer:

Let the cost of 1 kg of apples be Rs *x*.

And, cost of 1 kg of grapes = Rs *y*

According to the question, the algebraic representation is

$2x + y = 160$

$4x + 2y = 300$

For $2x + y = 160$,

$y = 160 - 2x$

The solution table is

x	50	60	70
y	60	40	20

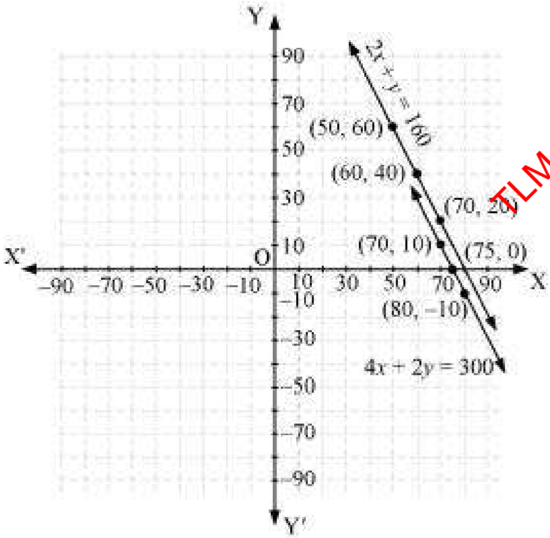
For $4x + 2y = 300$,

$$y = \frac{300 - 4x}{2}$$

The solution table is

x	70	80	75
y	10	−10	0

The graphical representation is as follows.



Exercise 3.2

Question 1:

Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

Answer:

(i) Let the number of girls be x and the number of boys be y.

According to the question, the algebraic representation is

$x + y = 10$

$x - y = 4$

For $x + y = 10$,

$x = 10 - y$

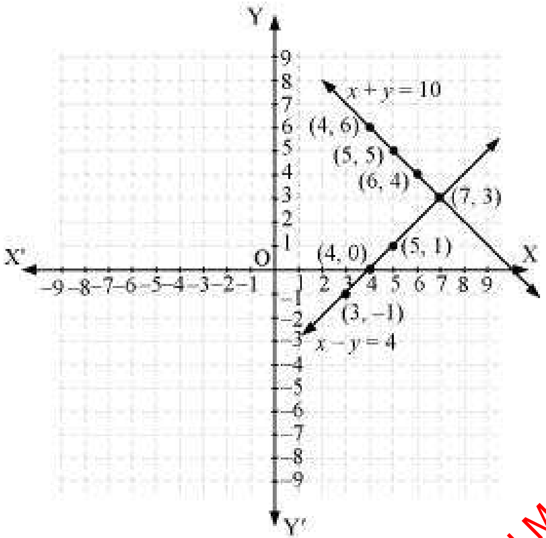
x	5	4	6
y	5	6	4

For $x - y = 4$,

$x = 4 + y$

x	5	4	3
y	1	0	−1

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (7, 3).

Therefore, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of 1 pencil be Rs x and the cost of 1 pen be Rs y.

According to the question, the algebraic representation is

$5x + 7y = 50$

$7x + 5y = 46$

For $5x + 7y = 50$,

$$x = \frac{50 - 7y}{5}$$

x	3	10	− 4
y	5	0	10

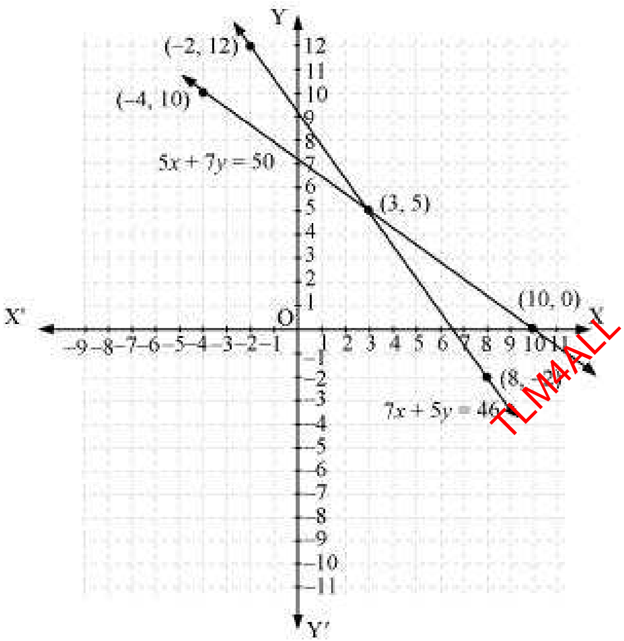
$7x + 5y = 46$

$$x = \frac{46 - 5y}{7}$$

x	8	3	− 2

y	− 2	5	12

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (3, 5).

Therefore, the cost of a pencil and a pen are Rs 3 and Rs 5 respectively.

Question 2:

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations at a point, are parallel or coincident:

$$\begin{array}{lll} \text{(i)}\ 5x-4y+8=0 & \text{(ii)}\ 9x+3y+12=0 & \text{(iii)}\ 6x-3y+10=0 \\ \quad 7x+6y-9=0 & \quad 18x+6y+24=0 & \quad 2x-y+9=0 \end{array}$$

Answer:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 5, \quad b_1 = -4, \quad c_1 = 8$

$a_2 = 7, \quad b_2 = 6, \quad c_2 = -9$

$\frac{a_1}{a_2} = \frac{5}{7}$

$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Hence, the lines representing the given pair of equations have a unique solution and the pair of lines intersects at exactly one point.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 9, \quad b_1 = 3, \quad c_1 = 12$

$a_2 = 18, \quad b_2 = 6, \quad c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, Hence, the lines representing the given pair of equations are coincident and there are infinite possible solutions for the given pair of equations.

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$a_1 = 6, \quad b_1 = -3, \quad c_1 = 10$

$a_2 = 2, \quad b_2 = -1, \quad c_2 = 9$

$$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}$$

$$\frac{c_1}{c_2} = \frac{10}{9}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Hence, the lines representing the given pair of equations are parallel to each other and hence, these lines will never intersect each other at any point or there is no possible solution for the given pair of equations.

Question 3:

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5; \quad 2x - 3y = 7$ (ii) $2x - 3y = 8; \quad 4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7; \quad 9x - 10y = 14$ (iv) $5x - 3y = 11; -10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8; \quad 2x + 3y = 12$

Answer:

(i) $3x + 2y = 5$

$2x - 3y = 7$

$$\frac{a_1}{a_2} = \frac{3}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{3}, \quad \frac{c_1}{c_2} = \frac{5}{7}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

These linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(ii) $2x - 3y = 8$

$4x - 6y = 9$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{8}{9}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

$9x - 10y = 14$

$$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}, \quad \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-1}{6}, \quad \frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(iv) $5x - 3y = 11$

$-10x + 6y = -22$

$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{11}{-22} = \frac{-1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

(v) $\frac{4}{3}x + 2y = 8$

$2x + 3y = 12$

$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

Question 4:

Which of the following pairs of linear equations are consistent/ inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5, \quad 2x + 2y = 10$

(ii) $x - y = 8, \quad 3x - 3y = 16$

(iii) $2x + y - 6 = 0, \quad 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Answer:

(i) $x + y = 5$

$2x + 2y = 10$

$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

$x + y = 5$

$x = 5 - y$

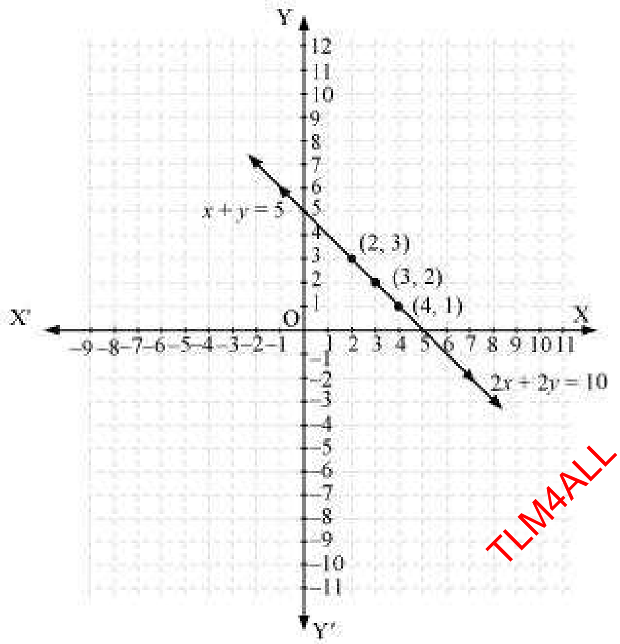
x	4	3	2
y	1	2	3

And, $2x + 2y = 10$

$x = \frac{10 - 2y}{2}$

x	4	3	2
y	1	2	3

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are overlapping each other. Therefore, infinite solutions are possible for the given pair of equations.

(ii)$x - y = 8$

$3x - 3y = 16$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii)$2x + y - 6 = 0$

$4x - 2y - 4 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

$2x + y - 6 = 0$

$y = 6 - 2x$

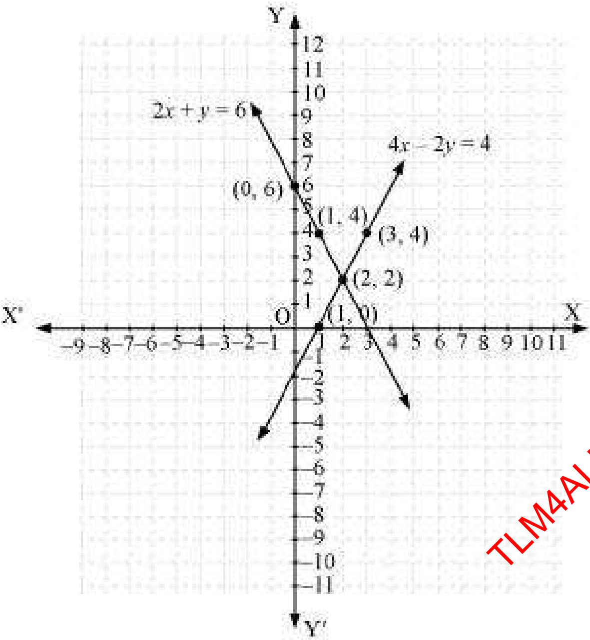
x	0	1	2
y	6	4	2

And $4x - 2y - 4 = 0$

$$y = \frac{4x-4}{2}$$

x	1	2	3
y	0	2	4

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at the only point i.e., (2, 2) and it is the solution for the given pair of equations.

(iv) $2x - 2y - 2 = 0$

$4x - 4y - 5 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{2}{5}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

Question 5:

Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Answer:

Let the width of the garden be x and length be y.

According to the question,

$y - x = 4$ (1)

$y + x = 36$ (2)

$y - x = 4$

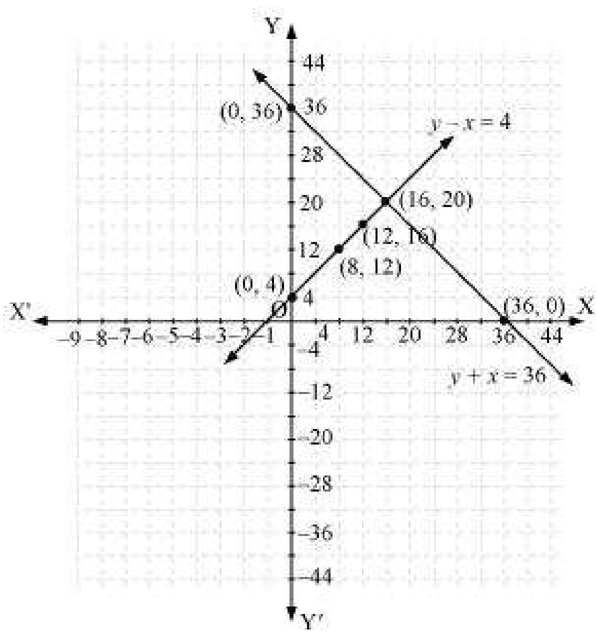
$y = x + 4$

x	0	8	12
y	4	12	16

$y + x = 36$

x	0	36	16
y	36	0	20

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at only point i.e., (16, 20). Therefore, the length and width of the given garden is 20 m and 16 m respectively.

Question 6:

Given the linear equation $2x + 3y - 8 = 0$, write another linear equations in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines (ii) parallel lines

(iii) coincident lines

Answer:

(i)Intersecting lines:

For this condition,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The second line such that it is intersecting the given line is $2x+4y-6=0$ as $\frac{a_1}{a_2}=\frac{2}{2}=1,\ \frac{b_1}{b_2}=\frac{3}{4}$ and $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

(ii) Parallel lines:

For this condition,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the second line can be

$4x + 6y - 8 = 0$

as $\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2},\quad \frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2},\ \frac{c_1}{c_2}=\frac{-8}{-8}=1$

And clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii)Coincident lines:

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the second line can be

$6x + 9y - 24 = 0$

$$\text{as } \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$$

And clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Question 7:

Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Answer:

$x - y + 1 = 0$

$x = y - 1$

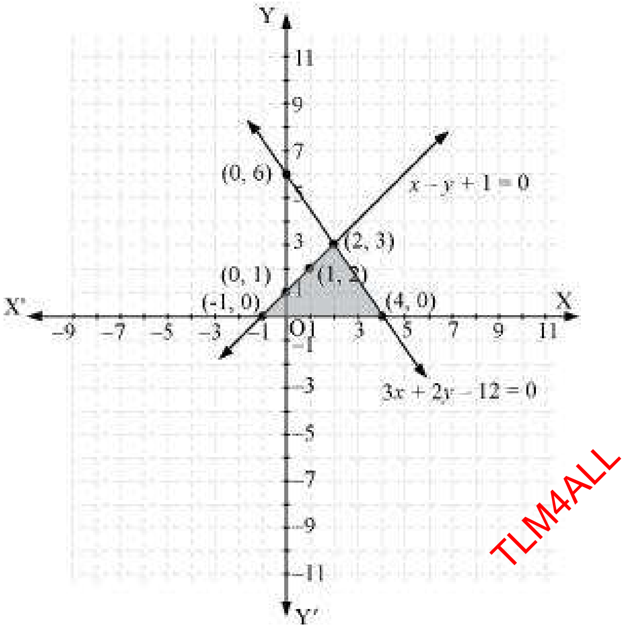
x	0	1	2
y	1	2	3

$3x + 2y - 12 = 0$

$$x = \frac{12 - 2y}{3}$$

x	4	2	0
y	0	3	6

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and x-axis at (−1, 0) and (4, 0). Therefore, the vertices of the triangle are (2, 3), (−1, 0), and (4, 0).

Exercise 3.3

Question 1:

Solve the following pair of linear equations by the substitution method.

(i) $x+y=14$

$x-y=4$

(ii) $s-t=3$

$\frac{s}{3}+\frac{t}{2}=6$

(iii) $3x-y=3$

$9x-3y=9$

(iv) $0.2x+0.3y=1.3$

$0.4x+0.5y=2.3$

(v) $\sqrt{2}x+\sqrt{3}y=0$

$\sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

Answer:

(i) $x + y = 14$ (1)

$x - y = 4$ (2)

From (1), we obtain

$x = 14 - y$ (3)

Substituting this value in equation (2), we obtain

$$(14-y)-y=4$$
$$14-2y=4$$
$$10=2y$$
$$y=5 \qquad (4)$$

Substituting this in equation (3), we obtain

$$x=9$$
$$\therefore x=9, y=5$$

(ii) $s-t=3 \qquad (1)$

$$\frac{s}{3}+\frac{t}{2}=6 \qquad (2)$$

From (1), we obtain

$s = t + 3 \qquad (3)$

Substituting this value in equation (2), we obtain

$\frac{t+3}{3} + \frac{t}{2} = 6$

$2t + 6 + 3t = 36$

$5t = 30$

$t = 6 \qquad (4)$

Substituting in equation (3), we obtain

$s = 9$

$\therefore s = 9, t = 6$

(iii)$3x - y = 3$ (1)

$9x - 3y = 9$ (2)

From (1), we obtain

$y = 3x - 3$ (3)

Substituting this value in equation (2), we obtain

$9x - 3(3x - 3) = 9$

$9x - 9x + 9 = 9$

$9 = 9$

This is always true.

Hence, the given pair of equations has infinite possible solutions and the relation between these variables can be given by

$y = 3x - 3$

Therefore, one of its possible solutions is $x = 1$, $y = 0$.

(iv) $0.2x + 0.3y = 1.3 \qquad (1)$

$0.4x + 0.5y = 2.3 \qquad (2)$

From equation (1), we obtain

$x = \frac{1.3 - 0.3y}{0.2} \qquad (3)$

Substituting this value in equation (2), we obtain

$$0.4\left(\frac{1.3-0.3y}{0.2}\right)+0.5y=2.3$$

$$2.6-0.6y+0.5y=2.3$$

$$2.6-2.3=0.1y$$

$$0.3=0.1y$$

$$y=3 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x=\frac{1.3-0.3\times 3}{0.2}$$

$$=\frac{1.3-0.9}{0.2}=\frac{0.4}{0.2}=2$$

$$\therefore x=2,\ y=3$$

(v) $\sqrt{2}x+\sqrt{3}y=0 \qquad (1)$

$$\sqrt{3}x-\sqrt{8}y=0 \qquad (2)$$

From equation (1), we obtain

$$x=\frac{-\sqrt{3}y}{\sqrt{2}} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\sqrt{3}\left(-\frac{\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$$

$$-\frac{3y}{\sqrt{2}}-2\sqrt{2}y=0$$

$$y\left(-\frac{3}{\sqrt{2}}-2\sqrt{2}\right)=0$$

$$y=0 \qquad (4)$$

Substituting this value in equation (3), we obtain

$x = 0$

$\therefore x = 0,\ y = 0$

(vi) $\frac{3}{2}x - \frac{5}{3}y = -2 \quad (1)$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \quad (2)$

From equation (1), we obtain

$9x - 10y = -12$

$x = \frac{-12+10y}{9} \quad (3)$

Substituting this value in equation (2), we obtain

$$\frac{\frac{-12+10y}{9}}{3} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-12+10y}{27} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-24+20y+27y}{54} = \frac{13}{6}$$

$$47y = 117 + 24$$

$$47y = 141$$

$$y = 3 \quad (4)$$

Substituting this value in equation (3), we obtain

$$x = \frac{-12+10\times 3}{9} = \frac{18}{9} = 2$$

Hence, $x = 2$, $y = 3$

Question 2:

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Answer:

$$2x+3y=11 \qquad (1)$$

$$2x-4y=-24 \qquad (2)$$

From equation (1), we obtain

$$x=\frac{11-3y}{2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$2\left(\frac{11-3y}{2}\right)-4y=-24$$

$$11-3y-4y=-24$$

$$-7y=-35$$

$$y=5 \qquad (4)$$

Putting this value in equation (3), we obtain

$$x=\frac{11-3\times 5}{2}=-\frac{4}{2}=-2$$

Hence, x = −2, y = 5

Also,

$$y=mx+3$$

$$5=-2m+3$$

$$-2m=2$$

$$m=-1$$

Question 3:

Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km.

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Answer:

(i) Let the first number be *x* and the other number be *y* such that *y* > *x*.

According to the given information,

$y = 3x \qquad (1)$

$y - x = 26 \qquad (2)$

On substituting the value of *y* from equation (1) into equation (2), we obtain

$3x - x = 26$

$x = 13 \qquad (3)$

Substituting this in equation (1), we obtain

y = 39

Hence, the numbers are 13 and 39.

(ii) Let the larger angle be *x* and smaller angle be *y*.

We know that the sum of the measures of angles of a supplementary pair is always 180º.

According to the given information,

$x+y=180^\circ \quad (1)$

$x-y=18^\circ \quad (2)$

From (1), we obtain

$x = 180^\circ - y$ (3)

Substituting this in equation (2), we obtain

$180^\circ - y - y = 18^\circ$

$162^\circ = 2y$

$81^\circ = y \quad (4)$

Putting this in equation (3), we obtain

$x = 180^\circ - 81^\circ$

$= 99^\circ$

Hence, the angles are 99º and 81º.

(iii) Let the cost of a bat and a ball be *x* and *y* respectively.

According to the given information,

$7x+6y=3800 \quad (1)$

$3x+5y=1750 \quad (2)$

From (1), we obtain

$$y=\frac{3800-7x}{6} \quad (3)$$

Substituting this value in equation (2), we obtain

$$3x+5\left(\frac{3800-7x}{6}\right)=1750$$

$$3x+\frac{9500}{3}-\frac{35x}{6}=1750$$

$$3x-\frac{35x}{6}=1750-\frac{9500}{3}$$

$$\frac{18x-35x}{6}=\frac{5250-9500}{3}$$
$$-\frac{17x}{6}=\frac{-4250}{3}$$
$$-17x=-8500$$
$$x=500 \qquad (4)$$

Substituting this in equation (3), we obtain

$$y=\frac{3800-7\times 500}{6}$$
$$=\frac{300}{6}=50$$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

(iv)Let the fixed charge be Rs x and per km charge be Rs y.

According to the given information,

$$x+10y=105 \qquad (1)$$
$$x+15y=155 \qquad (2)$$

From (3), we obtain

$$x=105-10y \qquad (3)$$

Substituting this in equation (2), we obtain

$$105-10y+15y=155$$
$$5y=50$$
$$y=10 \qquad (4)$$

Putting this in equation (3), we obtain

$$x=105-10\times 10$$
$$x=5$$

Hence, fixed charge = Rs 5

And per km charge = Rs 10

Charge for 25 km = x + 25y

= 5 + 250 = Rs 255

(v) Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x+2}{y+2}=\frac{9}{11}$$

$$11x+22=9y+18$$

$$11x-9y=-4 \qquad (1)$$

$$\frac{x+3}{y+3}=\frac{5}{6}$$

$$6x+18=5y+15$$

$$6x-5y=-3 \qquad (2)$$

From equation (1), we obtain $x=\frac{-4+9y}{11} \qquad (3)$

Substituting this in equation (2), we obtain

$$6\left(\frac{-4+9y}{11}\right)-5y=-3$$

$$-24+54y-55y=-33$$

$$-y=-9$$

$$y=9 \qquad (4)$$

Substituting this in equation (3), we obtain

$$x=\frac{-4+81}{11}=7$$

Hence, the fraction is $\frac{7}{9}$.

(vi) Let the age of Jacob be x and the age of his son be y.

According to the given information,

$$(x+5)=3(y+5)$$
$$x-3y=10 \qquad (1)$$
$$(x-5)=7(y-5)$$
$$x-7y=-30 \qquad (2)$$

From (1), we obtain

$$x=3y+10 \qquad (3)$$

Substituting this value in equation (2), we obtain

$$3y+10-7y=-30$$
$$-4y=-40$$
$$y=10 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x=3\times10+10$$
$$=40$$

Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years.

Exercise 3.4

Question 1:

Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $x+y=5$ and $2x-3y=4$ (ii) $3x+4y=10$ and $2x-2y=2$

(iii) $3x-5y-4=0$ and $9x=2y+7$ (iv) $\frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$

Answer:

(i) ***By elimination method***

$x+y=5 \quad (1)$

$2x-3y=4 \quad (2)$

Multiplying equation (1) by 2, we obtain

$2x+2y=10 \quad (3)$

Subtracting equation (2) from equation (3), we obtain

$5y=6$

$y=\frac{6}{5} \quad (4)$

Substituting the value in equation (1), we obtain

$x=5-\frac{6}{5}=\frac{19}{5}$

$\therefore x=\frac{19}{5}, y=\frac{6}{5}$

By substitution method

From equation (1), we obtain

$x=5-y$ (5)

Putting this value in equation (2), we obtain

$2(5-y)-3y=4$

$-5y=-6$

$y = \frac{6}{5}$

Substituting the value in equation (5), we obtain

$x = 5 - \frac{6}{5} = \frac{19}{5}$

$\therefore x = \frac{19}{5}, y = \frac{6}{5}$

(ii) ***By elimination method***

$3x + 4y = 10 \quad (1)$

$2x - 2y = 2 \quad (2)$

Multiplying equation (2) by 2, we obtain

$4x - 4y = 4 \quad (3)$

Adding equation (1) and (3), we obtain

$7x = 14$

$x = 2 \quad (4)$

Substituting in equation (1), we obtain

$6 + 4y = 10$

$4y = 4$

$y = 1$

Hence, $x = 2$, $y = 1$

By substitution method

From equation (2), we obtain

$x = 1 + y$ (5)

Putting this value in equation (1), we obtain

$3(1 + y) + 4y = 10$

$7y = 7$

$y = 1$

Substituting the value in equation (5), we obtain

$x = 1+1 = 2$

$\therefore x = 2, y = 1$

(iii) ***By elimination method***

$3x - 5y - 4 = 0 \qquad (1)$

$9x = 2y + 7$

$9x - 2y - 7 = 0 \qquad (2)$

Multiplying equation (1) by 3, we obtain

$9x - 15y - 12 = 0 \qquad (3)$

Subtracting equation (3) from equation (2), we obtain

$13y = -5$

$y = \dfrac{-5}{13} \qquad (4)$

Substituting in equation (1), we obtain

$3x + \dfrac{25}{13} - 4 = 0$

$3x = \dfrac{27}{13}$

$x = \dfrac{9}{13}$

$\therefore \quad x = \dfrac{9}{13}, y = \dfrac{-5}{13}$

By substitution method

From equation (1), we obtain

$x = \dfrac{5y+4}{3} \qquad (5)$

Putting this value in equation (2), we obtain

$$9\left(\frac{5y+4}{3}\right)-2y-7=0$$

$$13y=-5$$

$$y=-\frac{5}{13}$$

Substituting the value in equation (5), we obtain

$$x=\frac{5\left(\frac{-5}{13}\right)+4}{3}$$

$$x=\frac{9}{13}$$

$$\therefore \quad x=\frac{9}{13}, y=\frac{-5}{13}$$

(iv)***By elimination method***

$$\frac{x}{2}+\frac{2y}{3}=-1$$

$$3x+4y=-6 \quad (1)$$

$$x-\frac{y}{3}=3$$

$$3x-y=9 \qquad (2)$$

Subtracting equation (2) from equation (1), we obtain

$$5y=-15$$

$$y=-3 \qquad (3)$$

Substituting this value in equation (1), we obtain

$$3x-12=-6$$

$$3x=6$$

$$x=2$$

Hence, $x = 2$, $y = -3$

By substitution method

From equation (2), we obtain

$x = \frac{y+9}{3}$ (5)

Putting this value in equation (1), we obtain

$3\left(\frac{y+9}{3}\right) + 4y = -6$

$5y = -15$

$y = -3$

Substituting the value in equation (5), we obtain

$x = \frac{-3+9}{3} = 2$

$\therefore x = 2, y = -3$

Question 2:

Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Answer:

(i)Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x+1}{y-1}=1 \qquad \Rightarrow x-y=-2 \qquad (1)$$

$$\frac{x}{y+1}=\frac{1}{2} \qquad \Rightarrow 2x-y=1 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

x = 3 (3)

Substituting this value in equation (1), we obtain

$$3-y=-2$$
$$-y=-5$$
$$y=5$$

Hence, the fraction is $\frac{3}{5}$.

(ii)Let present age of Nuri = x

and present age of Sonu = y

According to the given information,

$$(x-5)=3(y-5)$$
$$x-3y=-10 \qquad (1)$$
$$(x+10)=2(y+10)$$
$$x-2y=10 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

y = 20 (3)

Substituting it in equation (1), we obtain

$x - 60 = -10$

$x = 50$

Hence, age of Nuri = 50 years

And, age of Sonu = 20 years

(iii)Let the unit digit and tens digits of the number be x and y respectively. Then, number = $10y + x$

Number after reversing the digits = $10x + y$

According to the given information,

$x + y = 9$ (1)

$9(10y + x) = 2(10x + y)$

$88y - 11x = 0$

$-x + 8y = 0$ (2)

Adding equation (1) and (2), we obtain

$9y = 9$

$y = 1$ (3)

Substituting the value in equation (1), we obtain

$x = 8$

Hence, the number is $10y + x = 10 \times 1 + 8 = 18$

(iv)Let the number of Rs 50 notes and Rs 100 notes be x and y respectively.

According to the given information,

$x + y = 25 \quad (1)$

$50x + 100y = 2000 \quad (2)$

Multiplying equation (1) by 50, we obtain

$50x + 50y = 1250 \quad (3)$

Subtracting equation (3) from equation (2), we obtain

$50y = 750$

$y = 15$

Substituting in equation (1), we have $x = 10$

Hence, Meena has 10 notes of Rs 50 and 15 notes of Rs 100 (v)Let the fixed charge for first three days and each day charge thereafter be Rs *x* and Rs *y* respectively.

According to the given information,

$x+4y=27 \qquad (1)$

$x+2y=21 \qquad (2)$

Subtracting equation (2) from equation (1), we obtain

$2y=6$

$y=3 \qquad (3)$

Substituting in equation (1), we obtain

$x+12=27$

$x=15$

Hence, fixed charge = Rs 15

And Charge per day = Rs 3

Exercise 3.5

Question 1:

Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x-3y-3=0$
$3x-9y-2=0$

(ii) $2x+y=5$
$3x+2y=8$

(iii) $3x-5y=20$
$6x-10y=40$

(iv) $x-3y-7=0$
$3x-3y-15=0$

Answer:

(i) $x-3y-3=0$

$3x-9y-2=0$

$$\frac{a_1}{a_2}=\frac{1}{3},\quad \frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\quad \frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

(ii) $2x+y=5$

$3x+2y=8$

$$\frac{a_1}{a_2}=\frac{2}{3},\quad \frac{b_1}{b_2}=\frac{1}{2},\quad \frac{c_1}{c_2}=\frac{-5}{-8}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$
$$\frac{x}{-8-(-10)}=\frac{y}{-15+16}=\frac{1}{4-3}$$
$$\frac{x}{2}=\frac{y}{1}=1$$
$$\frac{x}{2}=1,\quad \frac{y}{1}=1$$
$$x=2,\quad y=1$$

$\therefore$ $x = 2$, $y = 1$

(iii) $3x-5y=20$

$6x-10y=40$

$$\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2},\quad \frac{b_1}{b_2}=\frac{-5}{-10}=\frac{1}{2},\quad \frac{c_1}{c_2}=\frac{-20}{-40}=\frac{1}{2}$$
$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

(iv) $x-3y-7=0$

$3x-3y-15=0$

$$\frac{a_1}{a_2}=\frac{1}{3},\quad \frac{b_1}{b_2}=\frac{-3}{-3}=1,\quad \frac{c_1}{c_2}=\frac{-7}{-15}=\frac{7}{15}$$
$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

$$\frac{x}{45-(21)}=\frac{y}{-21-(-15)}=\frac{1}{-3-(-9)}$$

$$\frac{x}{24}=\frac{y}{-6}=\frac{1}{6}$$

$$\frac{x}{24}=\frac{1}{6} \text{ and } \frac{y}{-6}=\frac{1}{6}$$

$$x=4 \text{ and } y=-1$$

$$\therefore \; x=4, y=-1$$

Question 2:

(i) For which values of *a* and *b* will the following pair of linear equations have an infinite number of solutions?

$$2x+3y=7$$

$$(a-b)x+(a+b)y=3a+b-2$$

(ii) For which value of *k* will the following pair of linear equations have no solution?

$$3x+y=1$$

$$(2k-1)x+(k-1)y=2k+1$$

Answer:

(i) $2x+3y-7=0$

$$(a-b)x+(a+b)y-(3a+b-2)=0$$

$$\frac{a_1}{a_2}=\frac{2}{a-b}, \quad \frac{b_1}{b_2}=\frac{3}{a+b}, \quad \frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{7}{(3a+b-2)}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

$$6a+2b-4 = 7a-7b$$

$$a-9b = -4 \qquad (1)$$

$$\frac{2}{a-b} = \frac{3}{a+b}$$

$$2a+2b = 3a-3b$$

$$a-5b = 0 \qquad (2)$$

Subtracting (1) from (2), we obtain

$$4b = 4$$

$$b = 1$$

Substituting this in equation (2), we obtain

$$a - 5 \times 1 = 0$$

$$a = 5$$

Hence, a = 5 and b = 1 are the values for which the given equations give infinitely many solutions.

(ii) $3x + y - 1 = 0$

$$(2k-1)x + (k-1)y - 2k - 1 = 0$$

$$\frac{a_1}{a_2} = \frac{3}{2k-1}, \quad \frac{b_1}{b_2} = \frac{1}{k-1}, \quad \frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$$3k-3 = 2k-1$$

$k = 2$

Hence, for k = 2, the given equation has no solution.

Question 3:

Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$8x + 5y = 9$

$3x + 2y = 4$

Answer:

$8x + 5y = 9 \qquad (i)$

$3x + 2y = 4 \qquad (ii)$

From equation (ii), we obtain

$$x = \frac{4 - 2y}{3} \qquad (iii)$$

Substituting this value in equation (i), we obtain

$$8\left(\frac{4-2y}{3}\right) + 5y = 9$$

$32 - 16y + 15y = 27$

$-y = -5$

$y = 5 \qquad (iv)$

Substituting this value in equation (ii), we obtain

$3x + 10 = 4$

$x = -2$

Hence, $x = -2, y = 5$

Again, by cross-multiplication method, we obtain

$8x+5y-9=0$

$3x+2y-4=0$

$$\frac{x}{-20-(-18)}=\frac{y}{-27-(-32)}=\frac{1}{16-15}$$

$$\frac{x}{-2}=\frac{y}{5}=\frac{1}{1}$$

$$\frac{x}{-2}=1 \text{ and } \frac{y}{5}=1$$

$x=-2$ and $y=5$

Question 4:

Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i)A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii)A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii)Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v)The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer:

(i)Let x be the fixed charge of the food and y be the charge for food per day.

According to the given information,

$$x+20y=1000 \qquad (1)$$
$$x+26y=1180 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$6y=180$$
$$y=30$$

Substituting this value in equation (1), we obtain

$$x+20\times30=1000$$
$$x=1000-600$$
$$x=400$$

Hence, fixed charge = Rs 400

And charge per day = Rs 30

(ii)Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x-1}{y}=\frac{1}{3} \quad \Rightarrow \quad 3x-y=3 \qquad (1)$$
$$\frac{x}{y+8}=\frac{1}{4} \quad \Rightarrow \quad 4x-y=8 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$x=5 \qquad (3)$$

Putting this value in equation (1), we obtain

$$15-y=3$$
$$y=12$$

Hence, the fraction is $\frac{5}{12}$.

(iii)Let the number of right answers and wrong answers be *x* and *y* respectively.

According to the given information,

$$3x - y = 40 \qquad (1)$$
$$4x - 2y = 50$$
$$\Rightarrow 2x - y = 25 \qquad (2)$$

Subtracting equation (2) from equation (1), we obtain

x = 15 (3)

Substituting this in equation (2), we obtain

$$30 - y = 25$$
$$y = 5$$

Therefore, number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv)Let the speed of 1st car and 2nd car be *u* km/h and *v* km/h.

Respective speed of both cars while they are travelling in same direction = ($u - v$) km/h

Respective speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = ($u + v$) km/h

According to the given information,

$$5(u - v) = 100$$
$$\Rightarrow u - v = 20 \quad ...(1)$$
$$1(u + v) = 100 \quad ...(2)$$

Adding both the equations, we obtain

$$2u = 120$$
$$u = 60 \text{ km/h} \qquad (3)$$

Substituting this value in equation (2), we obtain

v = 40 km/h

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h

(v) Let length and breadth of rectangle be *x* unit and *y* unit respectively.

Area = *xy*

According to the question,

$$(x-5)(y+3) = xy-9$$

$$\Rightarrow 3x-5y-6=0 \qquad (1)$$

$$(x+3)(y+2) = xy+67$$

$$\Rightarrow 2x+3y-61=0 \qquad (2)$$

By cross-multiplication method, we obtain

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$x = 17, y = 9$$

Hence, the length and breadth of the rectangle are 17 units and 9 units respectively.

Exercise 3.6

Question 1:

Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$, $\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$, $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14$, $\frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$, $\frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5$, $\frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy$, $2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$, $\frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$, $\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Answer:

(i)
$$\frac{1}{2x}+\frac{1}{3y}=2$$
$$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations change as follows.

$$\frac{p}{2}+\frac{q}{3}=2 \quad \Rightarrow \quad 3p+2q-12=0 \quad (1)$$

$$\frac{p}{3}+\frac{q}{2}=\frac{13}{6} \quad \Rightarrow \quad 2p+3q-13=0 \quad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$$\frac{p}{10}=\frac{1}{5} \text{ and } \frac{q}{15}=\frac{1}{5}$$

$$p=2 \text{ and } q=3$$

$$\frac{1}{x}=2 \text{ and } \frac{1}{y}=3$$

$$x=\frac{1}{2} \text{ and } y=\frac{1}{3}$$

(ii) $$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$

Putting $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$ in the given equations, we obtain

$$2p+3q=2 \quad (1)$$

$$4p-9q=-1 \quad (2)$$

Multiplying equation (1) by 3, we obtain

$6p + 9q = 6$ (3)

Adding equation (2) and (3), we obtain

$10p = 5$

$p = \frac{1}{2} \qquad (4)$

Putting in equation (1), we obtain

$2 \times \frac{1}{2} + 3q = 2$

$3q = 1$

$q = \frac{1}{3}$

$p = \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2$

$x = 4$

and $q = \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{y} = 3$

$y = 9$

Hence, $x = 4, y = 9$

(iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Substituting $\frac{1}{x} = p$ in the given equations, we obtain

$4p + 3y = 14 \quad \Rightarrow \quad 4p + 3y - 14 = 0 \qquad (1)$

$3p - 4y = 23 \quad \Rightarrow \quad 3p - 4y - 23 = 0 \qquad (2)$

By cross-multiplication, we obtain

$$\frac{p}{-69-56}=\frac{y}{-42-(-92)}=\frac{1}{-16-9}$$

$$\frac{p}{-125}=\frac{y}{50}=\frac{-1}{25}$$

$$\frac{p}{-125}=\frac{-1}{25} \text{ and } \frac{y}{50}=\frac{-1}{25}$$

$p=5$ and $y=-2$

$$p=\frac{1}{x}=5$$

$$x=\frac{1}{5}$$

$$y=-2$$

(iv) $$\frac{5}{x-1}+\frac{1}{y-2}=2$$

$$\frac{6}{x-1}-\frac{3}{y-2}=1$$

Putting $\frac{1}{x-1}=p$ and $\frac{1}{y-2}=q$ in the given equation, we obtain

$5p+q=2 \qquad (1)$

$6p-3q=1 \qquad (2)$

Multiplying equation (1) by 3, we obtain

$15p+3q=6 \qquad (3)$

Adding (2) an (3), we obtain

$21p=7$

$$p=\frac{1}{3}$$

Putting this value in equation (1), we obtain

$$5\times\frac{1}{3}+q=2$$
$$q=2-\frac{5}{3}=\frac{1}{3}$$
$$p=\frac{1}{x-1}=\frac{1}{3}$$
$$\Rightarrow x-1=3$$
$$\Rightarrow x=4$$
$$q=\frac{1}{y-2}=\frac{1}{3}$$
$$y-2=3$$
$$y=5$$
$$\therefore x=4, y=5$$

(v) $$\frac{7x-2y}{xy}=5$$
$$\frac{7}{y}-\frac{2}{x}=5 \qquad (1)$$
$$\frac{8x+7y}{xy}=15$$
$$\frac{8}{y}+\frac{7}{x}=15 \qquad (2)$$

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in the given equation, we obtain

$$-2p+7q=5 \quad \Rightarrow \quad -2p+7q-5=0 \qquad (3)$$
$$7p+8q=15 \quad \Rightarrow \quad 7p+8q-15=0 \qquad (4)$$

By cross-multiplication method, we obtain

$$\frac{p}{-105-(-40)}=\frac{q}{-35-30}=\frac{1}{-16-49}$$

$$\frac{p}{-65}=\frac{q}{-65}=\frac{1}{-65}$$

$$\frac{p}{-65}=\frac{1}{-65} \text{ and } \frac{q}{-65}=\frac{1}{-65}$$

$$p=1 \quad \text{and} \quad q=1$$

$$p=\frac{1}{x}=1 \qquad q=\frac{1}{y}=1$$

$$x=1 \qquad y=1$$

(vi) $6x+3y=6xy$

$$\Rightarrow \frac{6}{y}+\frac{3}{x}=6 \qquad (1)$$

$$2x+4y=5xy$$

$$\frac{2}{y}+\frac{4}{x}=5 \qquad (2)$$

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in these equations, we obtain

$$3p+6q-6=0$$

$$4p+2q-5=0$$

By cross-multiplication method, we obtain

$$\frac{p}{-30-(-12)}=\frac{q}{-24-(-15)}=\frac{1}{6-24}$$

$$\frac{p}{-18}=\frac{q}{-9}=\frac{1}{-18}$$

$$\frac{p}{-18}=\frac{1}{-18} \text{ and } \frac{q}{-9}=\frac{1}{-18}$$

$$p=1 \quad \text{and} \quad q=\frac{1}{2}$$

$$p=\frac{1}{x}=1 \qquad q=\frac{1}{y}=\frac{1}{2}$$

$$x=1 \qquad\qquad y=2$$

Hence, $x = 1$, $y = 2$

(vii) $$\frac{10}{x+y}+\frac{2}{x-y}=4$$

$$\frac{15}{x+y}-\frac{5}{x-y}=-2$$

Putting $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$ in the given equations, we obtain

$$10p+2q=4 \quad \Rightarrow \quad 10p+2q-4=0 \quad (1)$$

$$15p-5q=-2 \quad \Rightarrow \quad 15p-5q+2=0 \quad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{4-20}=\frac{q}{-60-(20)}=\frac{1}{-50-30}$$

$$\frac{p}{-16}=\frac{q}{-80}=\frac{1}{-80}$$

$$\frac{p}{-16}=\frac{1}{-80} \text{ and } \frac{q}{-80}=\frac{1}{-80}$$

$$p=\frac{1}{5} \text{ and } q=1$$

$p = \dfrac{1}{x+y} = \dfrac{1}{5}$ and $q = \dfrac{1}{x-y} = 1$

$x + y = 5 \qquad (3)$

and $x - y = 1 \qquad (4)$

Adding equation (3) and (4), we obtain

$2x = 6$

$x = 3 \qquad (5)$

Substituting in equation (3), we obtain

y = 2

Hence, x = 3, y = 2

(viii) $\dfrac{1}{3x+y} + \dfrac{1}{3x-y} = \dfrac{3}{4}$

$\dfrac{1}{2(3x+y)} - \dfrac{1}{2(3x-y)} = \dfrac{-1}{8}$

Putting $\dfrac{1}{3x+y} = p$ and $\dfrac{1}{3x-y} = q$ in these equations, we obtain

$p + q = \dfrac{3}{4} \qquad (1)$

$\dfrac{p}{2} - \dfrac{q}{2} = \dfrac{-1}{8}$

$p - q = \dfrac{-1}{4} \qquad (2)$

Adding (1) and (2), we obtain

$2p = \dfrac{3}{4} - \dfrac{1}{4}$

$2p = \dfrac{1}{2}$

$p = \dfrac{1}{4}$

$$\frac{1}{4}-q=\frac{-1}{4}$$

$$q=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

$$p=\frac{1}{3x+y}=\frac{1}{4}$$

$$3x+y=4 \qquad (3)$$

$$q=\frac{1}{3x-y}=\frac{1}{2}$$

$$3x-y=2 \qquad (4)$$

Adding equations (3) and (4), we obtain

$$6x=6$$

$$x=1 \qquad (5)$$

Substituting in (3), we obtain

$$3(1)+y=4$$

$$y=1$$

Hence, $x = 1$, $y = 1$

Question 2:

Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer:

(i)Let the speed of Ritu in still water and the speed of stream be *x* km/h and *y* km/h respectively.

Speed of Ritu while rowing

Upstream = $(x-y)$ km/h

Downstream = $(x+y)$ km/h

According to question,

$2(x+y)=20$

$\Rightarrow x+y=10 \qquad (1)$

$2(x-y)=4$

$\Rightarrow x-y=2 \qquad (2)$

Adding equation (1) and (2), we obtain

$2x=12 \Rightarrow x=6$

Putting this in equation (1), we obtain

y = 4

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

(ii)Let the number of days taken by a woman and a man be *x* and *y* respectively.

Therefore, work done by a woman in 1 day = $\frac{1}{x}$

Work done by a man in 1 day = $\frac{1}{y}$

According to the question,

$$4\left(\frac{2}{x}+\frac{5}{y}\right)=1$$

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$$

$$3\left(\frac{3}{x}+\frac{6}{y}\right)=1$$

$$\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$$

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in these equations, we obtain

$$2p+5q=\frac{1}{4}$$

$$\Rightarrow \quad 8p+20q=1$$

$$3p+6q=\frac{1}{3}$$

$$\Rightarrow 9p+18q=1$$

By cross-multiplication, we obtain

$$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$$

$$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$$

$$\frac{p}{-2}=\frac{-1}{36} \text{ and } \frac{q}{-1}=\frac{1}{-36}$$

$$p=\frac{1}{18} \text{ and } q=\frac{1}{36}$$

$$p=\frac{1}{x}=\frac{1}{18} \text{ and } q=\frac{1}{y}=\frac{1}{36}$$

$$x=18 \qquad y=36$$

Hence, number of days taken by a woman = 18

Number of days taken by a man = 36

(iii) Let the speed of train and bus be *u* km/h and *v* km/h respectively.

According to the given information,

$$\frac{60}{u}+\frac{240}{v}=4 \qquad (1)$$

$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \qquad (2)$$

Putting $\frac{1}{u}=p$ and $\frac{1}{v}=q$ in these equations, we obtain

$$60p+240q=4 \qquad (3)$$

$$100p+200q=\frac{25}{6}$$

$$600p+1200q=25 \qquad (4)$$

Multiplying equation (3) by 10, we obtain

$$600p+2400q=40 \qquad (5)$$

Subtracting equation (4) from (5), we obtain

$$1200q=15$$

$$q=\frac{15}{1200}=\frac{1}{80} \qquad (6)$$

Substituting in equation (3), we obtain

$$60p+3=4$$

$$60p=1$$

$$p=\frac{1}{60}$$

$$p=\frac{1}{u}=\frac{1}{60} \quad \text{and} \quad q=\frac{1}{v}=\frac{1}{80}$$

$$u=60 \text{ km/h} \quad \text{and} \quad v=80 \text{ km/h}$$

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h

Exercise 3.7

Question 1:

The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Answer:

The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Let the age of Ani and Biju be x and y years respectively.

Therefore, age of Ani's father, Dharam = 2 × x = 2x years

And age of Biju's sister Cathy $= \frac{y}{2}$ years

By using the information given in the question,

Case (I) When Ani is older than Biju by 3 years,

$x - y = 3$ (i)

$$2x - \frac{y}{2} = 30$$

$4x - y = 60$ (ii)

Subtracting (i) from (ii), we obtain

$3x = 60 - 3 = 57$

$$x = \frac{57}{3} = 19$$

Therefore, age of Ani = 19 years

And age of Biju = 19 − 3 = 16 years

Case (II) When Biju is older than Ani,

$y - x = 3$ (i)

$$2x - \frac{y}{2} = 30$$

$4x - y = 60$ (*ii*)

Adding (*i*) and (*ii*), we obtain

$3x = 63$

$x = 21$

Therefore, age of Ani = 21 years

And age of Biju = 21 + 3 = 24 years

Question 2:

One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II)

[**Hint:** $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$]

Answer:

Let those friends were having Rs x and y with them.

Using the information given in the question, we obtain

$x + 100 = 2(y - 100)$

$x + 100 = 2y - 200$

$x - 2y = -300$ (*i*)

And, $6(x - 10) = (y + 10)$

$6x - 60 = y + 10$

$6x - y = 70$ (*ii*)

Multiplying equation (*ii*) by 2, we obtain

$12x - 2y = 140$ (*iii*)

Subtracting equation (*i*) from equation (*iii*), we obtain

$11x = 140 + 300$

$11x = 440$

$x = 40$

Using this in equation (*i*), we obtain

$40 - 2y = -300$

$40 + 300 = 2y$

$2y = 340$

$y = 170$

Therefore, those friends had Rs 40 and Rs 170 with them respectively.

Question 3:

A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Answer:

Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel was d km. We know that,

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$$

$$x = \frac{d}{t}$$

Or, $d = xt$ (i)

Using the information given in the question, we obtain

$$(x+10) = \frac{d}{(t-2)}$$

$$(x+10)(t-2) = d$$

$$xt + 10t - 2x - 20 = d$$

By using equation (i), we obtain

$-2x + 10t = 20$ (ii)

$$(x-10) = \frac{d}{(t+3)}$$

$$(x-10)(t+3) = d$$

$$xt - 10t + 3x - 30 = d$$

By using equation (i), we obtain

$3x - 10t = 30$ (iii)

Adding equations (ii) and (iii), we obtain

$x = 50$

Using equation (*ii*), we obtain

$(-2) \times (50) + 10t = 20$

$-100 + 10t = 20$

$10t = 120$

$t = 12$ hours

From equation (*i*), we obtain

Distance to travel = $d = xt$

$= 50 \times 12$

$= 600$ km

Hence, the distance covered by the train is 600 km.

Question 4:

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Answer:

Let the number of rows be x and number of students in a row be y.

Total students of the class

= Number of rows × Number of students in a row

$= xy$

Using the information given in the question,

Condition 1

Total number of students = $(x - 1)(y + 3)$

$xy = (x - 1)(y + 3) = xy - y + 3x - 3$

$3x - y - 3 = 0$

$3x - y = 3$ (*i*)

Condition 2

Total number of students = $(x + 2)(y - 3)$

$xy = xy + 2y - 3x - 6$

$3x - 2y = -6$ (*ii*)

Subtracting equation (*ii*) from (*i*),

$(3x - y) - (3x - 2y) = 3 - (-6)$

$-y + 2y = 3 + 6$

$y = 9$

By using equation (*i*), we obtain

$3x - 9 = 3$

$3x = 9 + 3 = 12$

$x = 4$

Number of rows $= x = 4$

Number of students in a row $= y = 9$

Number of total students in a class $= xy = 4 \times 9 = 36$

Question 5:

In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Answer:

Given that,

$\angle C = 3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2\angle A + 2\angle B$

$\angle B = 2\angle A$

$2\angle A - \angle B = 0 \ldots (i)$

We know that the sum of the measures of all angles of a triangle is 180°. Therefore,

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + 3\angle B = 180°$

$\angle A + 4\angle B = 180° \ldots (ii)$

Multiplying equation (*i*) by 4, we obtain

$8\angle A - 4\angle B = 0 \ldots (iii)$

Adding equations (*ii*) and (*iii*), we obtain

$9\angle A = 180°$

$\angle A = 20°$

From equation (*ii*), we obtain

$20° + 4\angle B = 180°$

$4\angle B = 160°$

$\angle B = 40°$

$\angle C = 3\angle B$

$= 3 \times 40° = 120°$

Therefore, $\angle A$, $\angle B$, $\angle C$ are 20°, 40°, and 120° respectively.

Question 6:

Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

Answer:

$5x - y = 5$

Or, $y = 5x - 5$

The solution table will be as follows.

x	0	1	2
y	−5	0	5

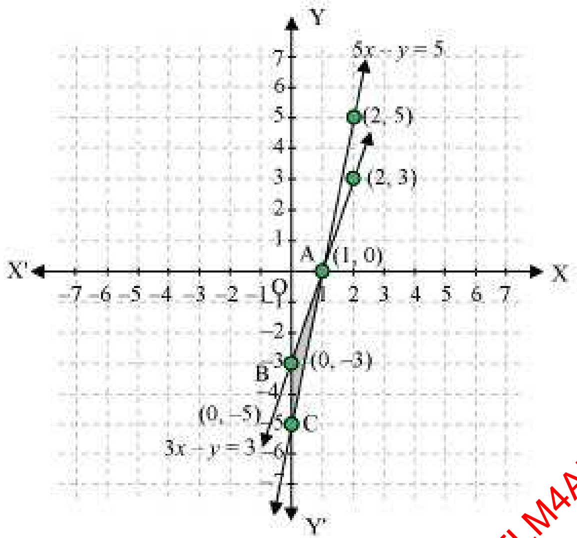
$3x - y = 3$

Or, $y = 3x - 3$

The solution table will be as follows.

x	0	1	2
y	− 3	0	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is ΔABC formed by these lines and *y*-axis.

The coordinates of vertices are A (1, 0), B (0, − 3), C (0, − 5).

Question 7:

Solve the following pair of linear equations.

(i) $px + qy = p - q$

$qx - py = p + q$

(ii) $ax + by = c$

$bx + ay = 1 + c$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)(x + y) = a^2 + b^2$

(v) $152x - 378y = -74$

$-378x + 152y = -604$

Answer:

(i) $px + qy = p - q$ ... (1)

$qx - py = p + q$ … (2)

Multiplying equation (1) by p and equation (2) by q, we obtain

$p^2x + pqy = p^2 - pq$ … (3)

$q^2x - pqy = pq + q^2$ … (4)

Adding equations (3) and (4), we obtain

$p^2x + q^2 x = p^2 + q^2$

$(p^2 + q^2) x = p^2 + q^2$

$$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$$

From equation (1), we obtain

$p (1) + qy = p - q$

$qy = - q$

$y = - 1$

(ii)$ax + by = c$ … (1)

$bx + ay = 1 + c$ … (2)

Multiplying equation (1) by a and equation (2) by b, we obtain

$a^2x + aby = ac$ … (3)

$b^2x + aby = b + bc$ … (4)

Subtracting equation (4) from equation (3),

$(a^2 - b^2) x = ac - bc - b$

$$x = \frac{c(a-b)-b}{a^2 - b^2}$$

From equation (1), we obtain

$ax + by = c$

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\}+by=c$$

$$\frac{ac(a-b)-ab}{a^2-b^2}+by=c$$

$$by=c-\frac{ac(a-b)-ab}{a^2-b^2}$$

$$by=\frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$by=\frac{abc-b^2c+ab}{a^2-b^2}$$

$$by=\frac{bc(a-b)+ab}{a^2-b^2}$$

$$y=\frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0$

Or, $bx - ay = 0$ … (1)

$ax + by = a^2 + b^2$ … (2)

Multiplying equation (1) and (2) by b and a respectively, we obtain

$b^2x - aby = 0$ … (3)

$a^2x + aby = a^3 + ab^2$ … (4)

Adding equations (3) and (4), we obtain

$b^2x + a^2x = a^3 + ab^2$

$x (b^2 + a^2) = a (a^2 + b^2)$

$x = a$

By using (1), we obtain

$b (a) - ay = 0$

$ab - ay = 0$

$ay = ab$

$y = b$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ ... (1)

$(a + b)(x + y) = a^2 + b^2$

$(a + b)x + (a + b)y = a^2 + b^2$ ... (2)

Subtracting equation (2) from (1), we obtain

$(a - b)x - (a + b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$

$(a - b - a - b)x = -2ab - 2b^2$

$-2bx = -2b(a + b)$

$x = a + b$

Using equation (1), we obtain

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$(a + b)y = -2ab$

$$y = \frac{-2ab}{a+b}$$

(v) $152x - 378y = -74$

$76x - 189y = -37$

$$x = \frac{189y - 37}{76} \quad \text{... (1)}$$

$-378x + 152y = -604$

$-189x + 76y = -302$ ... (2)

Substituting the value of x in equation (2), we obtain

$$-189\left(\frac{189y - 37}{76}\right) + 76y = -302$$

$-(189)^2 y + 189 \times 37 + (76)^2 y = -302 \times 76$

$189 \times 37 + 302 \times 76 = (189)^2 y - (76)^2 y$

$6993 + 22952 = (189 - 76)(189 + 76)y$

$29945 = (113)(265)y$

$y = 1$

From equation (1), we obtain

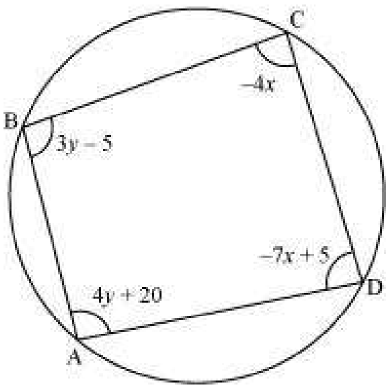
$$x = \frac{189(1) - 37}{76}$$

$$x = \frac{189 - 37}{76} = \frac{152}{76}$$

$$x = 2$$

Question 8:

ABCD is a cyclic quadrilateral finds the angles of the cyclic quadrilateral.



Answer:

We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.

Therefore, ∠A + ∠C = 180

$4y + 20 - 4x = 180$

$-4x + 4y = 160$

$x - y = -40$ (*i*)

Also, ∠B + ∠D = 180

$3y - 5 - 7x + 5 = 180$

$-7x + 3y = 180$ (*ii*)

Multiplying equation (*i*) by 3, we obtain

$3x - 3y = -120$ (*iii*)

Adding equations (*ii*) and (*iii*), we obtain

$-7x + 3x = 180 - 120$

$x = -15$

By using equation (*i*), we obtain

$x - y = -40$

$-15 - y = -40$

$y = -15 + 40 = 25$

$\angle A = 4y + 20 = 4(25) + 20 = 120°$

$\angle B = 3y - 5 = 3(25) - 5 = 70°$

$\angle C = -4x = -4(-15) = 60°$

$\angle D = -7x + 5 = -7(-15) + 5 = 110°$

Exercise 4.1

Question 1:

Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$

(ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$

(iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$

(vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Answer:

(i) $(x+1)^2 = 2(x-3) \Rightarrow x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3-x) \Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3) \Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow 3x - 1 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5) \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x \Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(v) $(2x-1)(x-3)=(x+5)(x-1)\Rightarrow 2x^2-7x+3=x^2+4x-5\Rightarrow x^2-11x+8=0$ It is of the form $ax^2+bx+c=0$.

Hence, the given equation is a quadratic equation.

(vi) $x^2+3x+1=(x-2)^2\Rightarrow x^2+3x+1=x^2+4-4x\Rightarrow 7x-3=0$

It is not of the form $ax^2+bx+c=0$.

Hence, the given equation is not a quadratic equation.

(vii) $(x+2)^3=2x(x^2-1)\Rightarrow x^3+8+6x^2+12x=2x^3-2x\Rightarrow x^3-14x-6x^2-8=0$ It is not of the form $ax^2+bx+c=0$.

Hence, the given equation is not a quadratic equation.

(viii) $x^3-4x^3-x+1=(x-2)^3\Rightarrow x^3-4x^2-x+1=x^3-8-6x^2+12x\Rightarrow 2x^2-13x+9=0$

It is of the form $ax^2+bx+c=0$.

Hence, the given equation is a quadratic equation.

Question 2:

Represent the following situations in the form of quadratic equations.

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Answer:

(i) Let the breadth of the plot be *x* m.

Hence, the length of the plot is (2*x* + 1) m.

Area of a rectangle = Length × Breadth

∴ 528 = *x* (2*x* + 1)

$\Rightarrow 2x^2 + x - 528 = 0$

(ii) Let the consecutive integers be *x* and *x* + 1.

It is given that their product is 306.

$\therefore \; x(x+1) = 306 \Rightarrow x^2 + x - 306 = 0$

(iii) Let Rohan's age be *x*.

Hence, his mother's age = *x* + 26

3 years hence,

Rohan's age = *x* + 3

Mother's age = *x* + 26 + 3 = *x* + 29

It is given that the product of their ages after 3 years is 360.

$\therefore (x+3)(x+29) = 360$

$\Rightarrow x^2 + 32x - 273 = 0$

(iv) Let the speed of train be *x* km/h.

Time taken to travel 480 km = $\frac{480}{x}$ hrs

In second condition, let the speed of train = $(x-8)$ km/h

It is also given that the train will take 3 hours to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\frac{480}{x}+3\right)$hrs

Speed × Time = Distance

$$(x-8)\left(\frac{480}{x}+3\right)=480$$

$$\Rightarrow 480+3x-\frac{3840}{x}-24=480$$

$$\Rightarrow 3x-\frac{3840}{x}=24$$

$$\Rightarrow 3x^2-24x+3840=0$$

$$\Rightarrow x^2-8x+1280=0$$

Exercise 4.2

Question 1:

Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$ (ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ (iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Answer:

(i) $x^2 - 3x - 10$

$= x^2 - 5x + 2x - 10$

$= x(x-5) + 2(x-5)$

$= (x-5)(x+2)$

Roots of this equation are the values for which $(x-5)(x+2) = 0$

$\therefore$ $x - 5 = 0$ or $x + 2 = 0$

i.e., $x = 5$ or $x = -2$

(ii) $2x^2 + x - 6$

$= 2x^2 + 4x - 3x - 6$

$= 2x(x+2) - 3(x+2)$

$= (x+2)(2x-3)$

Roots of this equation are the values for which $(x+2)(2x-3) = 0$

$\therefore$ $x + 2 = 0$ or $2x - 3 = 0$

i.e., $x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}$

$=\sqrt{2}x^2+5x+2x+5\sqrt{2}$

$=x\left(\sqrt{2}x+5\right)+\sqrt{2}\left(\sqrt{2}x+5\right)$

$=\left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)$

Roots of this equation are the values for which $\left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)$ = 0

$\therefore$ $\sqrt{2}x+5$ = 0 or $x+\sqrt{2}$ = 0

i.e., $x = \frac{-5}{\sqrt{2}}$ or $x = -\sqrt{2}$

(iv) $2x^2-x+\frac{1}{8}$

$=\frac{1}{8}\left(16x^2-8x+1\right)$

$=\frac{1}{8}\left(16x^2-4x-4x+1\right)$

$=\frac{1}{8}\left(4x\left(4x-1\right)-1\left(4x-1\right)\right)$

$=\frac{1}{8}\left(4x-1\right)^2$

Roots of this equation are the values for which $\left(4x-1\right)^2$ = 0

Therefore, $\left(4x-1\right)=0$ or $\left(4x-1\right)=0$

i.e., $x=\frac{1}{4}$ or $x=\frac{1}{4}$

(v) $100x^2 - 20x + 1$

$= 100x^2 - 10x - 10x + 1$

$= 10x(10x-1) - 1(10x-1)$

$= (10x-1)^2$

Roots of this equation are the values for which $(10x-1)^2 = 0$

Therefore, $(10x-1) = 0$ or $(10x-1) = 0$

i.e., $x = \frac{1}{10}$ or $x = \frac{1}{10}$

Question 2:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Answer:

(i) Let the number of John's marbles be x.

Therefore, number of Jivanti's marble = $45 - x$

After losing 5 marbles,

Number of John's marbles = $x - 5$

Number of Jivanti's marbles = $45 - x - 5 = 40 - x$

It is given that the product of their marbles is 124.

$\therefore (x-5)(40-x)=124$

$\Rightarrow x^2-45x+324=0$

$\Rightarrow x^2-36x-9x+324=0$

$\Rightarrow x(x-36)-9(x-36)=0$

$\Rightarrow (x-36)(x-9)=0$

Either $x-36 = 0$ or $x - 9 = 0$

i.e., $x = 36$ or $x = 9$

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = 45 − 36 = 9

If number of John's marbles = 9,

Then, number of Jivanti's marbles = 45 − 9 = 36

(ii) Let the number of toys produced be x.

$\therefore$ Cost of production of each toy = Rs $(55 - x)$

It is given that, total production of the toys = Rs 750

$\therefore x(55-x)=750$

$\Rightarrow x^2-55x+750=0$

$\Rightarrow x^2-25x-30x+750=0$

$\Rightarrow x(x-25)-30(x-25)=0$

$\Rightarrow (x-25)(x-30)=0$

Either $x-25 = 0$ or $x - 30 = 0$

i.e., $x = 25$ or $x = 30$

Hence, the number of toys will be either 25 or 30.

Question 4:

Which of the following are APs? If they form an A.P. find the common difference d and write three more terms.

(i) 2, 4, 8, 16 …

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

(iii) − 1.2, − 3.2, − 5.2, − 7.2 …

(iv) − 10, − 6, − 2, 2 …

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots$

(vi) 0.2, 0.22, 0.222, 0.2222 …

(vii) 0, − 4, − 8, − 12 …

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

(ix) 1, 3, 9, 27 …

(x) a, $2a$, $3a$, $4a$ …

(xi) a, a^2, a^3, a^4 …

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$

(xv) $1^2, 5^2, 7^2, 73 \ldots$

Answer:

(i) 2, 4, 8, 16 …

It can be observed that

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

i.e., $a_{k+1} - a_k$ is not the same every time. Therefore, the given numbers are not forming an A.P.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

It can be observed that

$$a_2 - a = \frac{5}{2} - 2 = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, $d = \frac{1}{2}$ and the given numbers are in A.P.

Three more terms are

$$a_5 = \frac{7}{2} + \frac{1}{2} = 4$$

$$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$$

$$a_7 = \frac{9}{2} + \frac{1}{2} = 5$$

(iii) −1.2, −3.2, −5.2, −7.2 …

It can be observed that

$a_2 - a_1 = (-3.2) - (-1.2) = -2$

$a_3 - a_2 = (-5.2) - (-3.2) = -2$

$a_4 - a_3 = (-7.2) - (-5.2) = -2$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = -2$

The given numbers are in A.P.

Three more terms are

$a_5 = -7.2 - 2 = -9.2$

$a_6 = -9.2 - 2 = -11.2$

$a_7 = -11.2 - 2 = -13.2$

(iv) −10, −6, −2, 2 …

It can be observed that

$a_2 - a_1 = (-6) - (-10) = 4$

$a_3 - a_2 = (-2) - (-6) = 4$

$a_4 - a_3 = (2) - (-2) = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = 4$

The given numbers are in A.P.

Three more terms are

$a_5 = 2 + 4 = 6$

$a_6 = 6 + 4 = 10$

$a_7 = 10 + 4 = 14$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

It can be observed that

$$a_2 - a_1 = 3+\sqrt{2} - 3 = \sqrt{2}$$
$$a_3 - a_2 = 3+2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$
$$a_4 - a_3 = 3+3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = \sqrt{2}$

The given numbers are in A.P.

Three more terms are

$a_5 = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$a_6 = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

$a_7 = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222 ….

It can be observed that

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(vii) 0, −4, −8, −12 …

It can be observed that

$a_2 - a_1 = (-4) - 0 = -4$

$a_3 - a_2 = (-8) - (-4) = -4$

$a_4 - a_3 = (-12) - (-8) = -4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = -4$

The given numbers are in A.P.

Three more terms are

$a_5 = -12 - 4 = -16$

$a_6 = -16 - 4 = -20$

$a_7 = -20 - 4 = -24$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

It can be observed that

$$a_2 - a_1 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$
$$a_3 - a_2 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$
$$a_4 - a_3 = \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = 0$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = 0$

The given numbers are in A.P.

Three more terms are

$$a_5 = -\frac{1}{2} - 0 = -\frac{1}{2}$$
$$a_6 = -\frac{1}{2} - 0 = -\frac{1}{2}$$
$$a_7 = -\frac{1}{2} - 0 = -\frac{1}{2}$$

(ix) 1, 3, 9, 27 …

It can be observed that

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_4 - a_3 = 27 - 9 = 18$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(x) a, $2a$, $3a$, $4a$ …

It can be observed that

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = a$

The given numbers are in A.P.

Three more terms are

$a_5 = 4a + a = 5a$

$a_6 = 5a + a = 6a$

$a_7 = 6a + a = 7a$

(xi) $a, a^2, a^3, a^4 \ldots$

It can be observed that

$a_2 - a_1 = a^2 - a = a(a - 1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a - 1)$

$a_4 - a_3 = a^4 - a^3 = a^3(a - 1)$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

It can be observed that

$$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$
$$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$
$$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, the given numbers are in A.P.

And, $d = \sqrt{2}$

Three more terms are

$a_5 = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

It can be observed that

$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3 \times 2} - \sqrt{3} = \sqrt{3}\left(\sqrt{2} - 1\right)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}\left(\sqrt{3} - \sqrt{2}\right)$

$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3 \times 3} = \sqrt{3}\left(2 - \sqrt{3}\right)$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$

Or, 1, 9, 25, 49 …..

It can be observed that

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

$a_4 - a_3 = 49 - 25 = 24$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(xv) $1^2, 5^2, 7^2, 73 \ldots$

Or 1, 25, 49, 73 …

It can be observed that

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

i.e., $a_{k+1} - a_k$ is same every time.

Therefore, the given numbers are in A.P.

And, $d = 24$

Three more terms are

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

$a_7 = 121 + 24 = 145$

Question 3:

Find two numbers whose sum is 27 and product is 182.

Answer:

Let the first number be x and the second number is $27 - x$.

Therefore, their product = $x(27 - x)$

It is given that the product of these numbers is 182.

Therefore, $x(27-x) = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x-13) - 14(x-13) = 0$

$\Rightarrow (x-13)(x-14) = 0$

Either $x - 13 = 0$ or $x - 14 = 0$

i.e., $x = 13$ or $x = 14$

If first number = 13, then

Other number = 27 − 13 = 14

If first number = 14, then

Other number = 27 − 14 = 13

Therefore, the numbers are 13 and 14.

Question 4:

Find two consecutive positive integers, sum of whose squares is 365.

Answer:

Let the consecutive positive integers be x and $x + 1$.

$$\text{Given that } x^2 + (x+1)^2 = 365$$
$$\Rightarrow x^2 + x^2 + 1 + 2x = 365$$
$$\Rightarrow 2x^2 + 2x - 364 = 0$$
$$\Rightarrow x^2 + x - 182 = 0$$
$$\Rightarrow x^2 + 14x - 13x - 182 = 0$$
$$\Rightarrow x(x+14) - 13(x+14) = 0$$
$$\Rightarrow (x+14)(x-13) = 0$$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$

Since the integers are positive, x can only be 13.

$\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

Question 5:

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer:

Let the base of the right triangle be x cm.

Its altitude = $(x - 7)$ cm

From pythagoras theorem,

$$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$$

$$\therefore x^2 + (x-7)^2 = 13^2$$

$$\Rightarrow x^2 + x^2 + 49 - 14x = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow x(x-12) + 5(x-12) = 0$$

$$\Rightarrow (x-12)(x+5) = 0$$

Either $x - 12 = 0$ or $x + 5 = 0$, i.e., $x = 12$ or $x = -5$

Since sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be (12 − 7) cm = 5 cm.

Question 6:

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Answer:

Let the number of articles produced be x.

Therefore, cost of production of each article = Rs $(2x + 3)$

It is given that the total production is Rs 90.

$$\therefore x(2x+3)=90$$
$$\Rightarrow 2x^2+3x-90=0$$
$$\Rightarrow 2x^2+15x-12x-90=0$$
$$\Rightarrow x(2x+15)-6(2x+15)=0$$
$$\Rightarrow (2x+15)(x-6)=0$$

Either $2x + 15 = 0$ or $x - 6 = 0$, i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = 2 × 6 + 3 = Rs 15

Exercise 4.3

Question 1:

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3x} + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Answer:

(i) $2x^2 - 7x + 3 = 0$

$\Rightarrow 2x^2 - 7x = -3$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$

$\Rightarrow x^2 - 2 \times x \times \frac{7}{4} = -\frac{3}{2}$

On adding $\left(\frac{7}{4}\right)^2$ to both sides of equation, we obtain

$\Rightarrow (x)^2 - 2 \times x \times \frac{7}{4} + \left(\frac{7}{4}\right)^2 = \left(\frac{7}{4}\right)^2 - \frac{3}{2}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$

$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow \left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$

$\Rightarrow x = \frac{7}{4} \pm \frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow x = \frac{12}{4}$ or $x = \frac{2}{4}$

$\Rightarrow x = 3$ or $\frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$

$\Rightarrow 2x^2 + x = 4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = 2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

$\Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\Rightarrow x = \pm\frac{\sqrt{33}}{4} - \frac{1}{4}$

$\Rightarrow x = \frac{\pm\sqrt{33} - 1}{4}$

$\Rightarrow x = \frac{\sqrt{33} - 1}{4}$ or $\frac{-\sqrt{33} - 1}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + \left(\sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x + \sqrt{3}\right) = 0$ and $\left(2x + \sqrt{3}\right) = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{2}$ and $x = \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

$\Rightarrow 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = -2$

$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} = -2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$

However, the square of a number cannot be negative.

Therefore, there is no real root for the given equation.

Question 2:

Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Answer:

(i) $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -7, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{25}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

$$\Rightarrow x = \frac{7+5}{4} \text{ or } \frac{7-5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 3 \text{ or } \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = 1, c = -4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\therefore x = \frac{-1 + \sqrt{33}}{4} \text{ or } \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2+4\sqrt{3}x+3=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain

$a=4, b=4\sqrt{3}, c=3$

By using quadratic formula, we obtain

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-4\sqrt{3}\pm\sqrt{48-48}}{8}$$

$$\Rightarrow x=\frac{-4\sqrt{3}\pm 0}{8}$$

$$\therefore\ x=\frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

(iv) $2x^2+x+4=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain

$a=2, b=1, c=4$

By using quadratic formula, we obtain

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{1-32}}{4}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{-31}}{4}$$

However, the square of a number cannot be negative.

Therefore, there is no real root for the given equation.

Question 3:

Find the roots of the following equations:

(i) $x-\frac{1}{x}=3, x\neq 0$ (ii) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x\neq -4, 7$

Answer:

(i) $x-\frac{1}{x}=3 \Rightarrow x^2-3x-1=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain

$a=1, b=-3, c=-1$

By using quadratic formula, we obtain

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{3\pm\sqrt{9+4}}{2}$$

$$\Rightarrow x=\frac{3\pm\sqrt{13}}{2}$$

Therefore, $x=\frac{3+\sqrt{13}}{2}$ or $\frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$

$$\Rightarrow \frac{x-7-x-4}{(x+4)(x-7)}=\frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x+4)(x-7)}=\frac{11}{30}$$

$$\Rightarrow (x+4)(x-7)=-30$$

$$\Rightarrow x^2-3x-28=-30$$

$$\Rightarrow x^2-3x+2=0$$

$$\Rightarrow x^2-2x-x+2=0$$

$$\Rightarrow x(x-2)-1(x-2)=0$$

$$\Rightarrow (x-2)(x-1)=0$$

$$\Rightarrow x=1 \text{ or } 2$$

Question 4:

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Answer:

Let the present age of Rehman be x years.

Three years ago, his age was $(x - 3)$ years.

Five years hence, his age will be $(x + 5)$ years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow 3(2x+2)=(x-3)(x+5)$$

$$\Rightarrow 6x+6=x^2+2x-15$$

$$\Rightarrow x^2-4x-21=0$$

$$\Rightarrow x^2-7x+3x-21=0$$

$$\Rightarrow x(x-7)+3(x-7)=0$$

$$\Rightarrow (x-7)(x+3)=0$$

$$\Rightarrow x=7,-3$$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

Question 5:

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Answer:

Let the marks in Maths be x.

Then, the marks in English will be $30 - x$.

According to the given question,

$$(x+2)(30-x-3)=210$$
$$(x+2)(27-x)=210$$
$$\Rightarrow -x^2+25x+54=210$$
$$\Rightarrow x^2-25x+156=0$$
$$\Rightarrow x^2-12x-13x+156=0$$
$$\Rightarrow x(x-12)-13(x-12)=0$$
$$\Rightarrow (x-12)(x-13)=0$$
$$\Rightarrow x=12,13$$

If the marks in Maths are 12, then marks in English will be 30 − 12 = 18

If the marks in Maths are 13, then marks in English will be 30 − 13 = 17

Question 6:

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Answer:

Let the shorter side of the rectangle be *x* m.

Then, larger side of the rectangle = (*x* + 30) m

Diagonal of the rectangle $= \sqrt{x^2 + (x+30)^2}$

It is given that the diagonal of the rectangle is 60 m more than the shorter side.

$$\therefore \sqrt{x^2 + (x+30)^2} = x + 60$$

$$\Rightarrow x^2 + (x+30)^2 = (x+60)^2$$

$$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$$

$$\Rightarrow x^2 - 60x - 2700 = 0$$

$$\Rightarrow x^2 - 90x + 30x - 2700 = 0$$

$$\Rightarrow x(x-90) + 30(x-90)$$

$$\Rightarrow (x-90)(x+30) = 0$$

$$\Rightarrow x = 90, -30$$

However, side cannot be negative. Therefore, the length of the shorter side will be

90 m.

Hence, length of the larger side will be (90 + 30) m = 120 m

Question 7:

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Answer:

Let the larger and smaller number be *x* and *y* respectively.

According to the given question,

$x^2 - y^2 = 180$ and $y^2 = 8x$

$\Rightarrow x^2 - 8x = 180$

$\Rightarrow x^2 - 8x - 180 = 0$

$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$\Rightarrow x(x-18) + 10(x-18) = 0$

$\Rightarrow (x-18)(x+10) = 0$

$\Rightarrow x = 18, -10$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.

Therefore, the larger number will be 18 only.

$x = 18$

$\therefore\ y^2 = 8x = 8 \times 18 = 144$

$\Rightarrow\ y = \pm\sqrt{144} = \pm 12$

$\therefore$ Smaller number $= \pm 12$

Therefore, the numbers are 18 and 12 or 18 and −12.

Question 8:

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Answer:

Let the speed of the train be *x* km/hr.

Time taken to cover 360 km $= \dfrac{360}{x}$ hr

According to the given question,

$$(x+5)\left(\frac{360}{x}-1\right)=360$$
$$\Rightarrow (x+5)\left(\frac{360}{x}-1\right)=360$$
$$\Rightarrow 360-x+\frac{1800}{x}-5=360$$
$$\Rightarrow x^2+5x-1800=0$$
$$\Rightarrow x^2+45x-40x-1800=0$$
$$\Rightarrow x(x+45)-40(x+45)=0$$
$$\Rightarrow (x+45)(x-40)=0$$
$$\Rightarrow \ x=40,-45$$

However, speed cannot be negative.

Therefore, the speed of train is 40 km/h

Question 9:

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Answer:

Let the time taken by the smaller pipe to fill the tank be *x* hr.

Time taken by the larger pipe = (*x* – 10) hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8}=\frac{75}{8}$ hours by both the pipes together. Therefore,

$$\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$$
$$\frac{x-10+x}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow \frac{2x-10}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow 75(2x-10)=8x^2-80x$$
$$\Rightarrow 150x-750=8x^2-80x$$
$$\Rightarrow 8x^2-230x+750=0$$
$$\Rightarrow 8x^2-200x-30x+750=0$$
$$\Rightarrow 8x(x-25)-30(x-25)=0$$
$$\Rightarrow (x-25)(8x-30)=0$$
$$i.e., x=25, \frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 − 10 =15 hours respectively.

Question 10:

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Answer:

Let the average speed of passenger train be x km/h.

Average speed of express train = (x + 11) km/h

It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x}-\frac{132}{x+11}=1$$

$$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right]=1$$

$$\Rightarrow \frac{132\times 11}{x(x+11)}=1$$

$$\Rightarrow 132\times 11=x(x+11)$$

$$\Rightarrow x^2+11x-1452=0$$

$$\Rightarrow x^2+44x-33x-1452=0$$

$$\Rightarrow x(x+44)-33(x+44)=0$$

$$\Rightarrow (x+44)(x-33)=0$$

$$\Rightarrow x=-44,33$$

Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

Question 11:

Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Answer:

Let the sides of the two squares be x m and y m. Therefore, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.

It is given that

$4x - 4y = 24$

$x - y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$

$\Rightarrow (6+y)^2 + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y+18) - 12(y+18) = 0$

$\Rightarrow (y+18)(y-12) = 0$

$\Rightarrow y = -18 \text{ or } 12.$

However, side of a square cannot be negative.

Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m

Exercise 4.4

Question 1:

Find the nature of the roots of the following quadratic equations.

If the real roots exist, find them;

(I) $2x^2 - 3x + 5 = 0$

(II) $3x^2 - 4\sqrt{3}x + 4 = 0$

(III) $2x^2 - 6x + 3 = 0$

Answer:

We know that for a quadratic equation $ax^2 + bx + c = 0$, discriminant is $b^2 - 4ac$.

(A) If $b^2 - 4ac > 0 \rightarrow$ two distinct real roots

(B) If $b^2 - 4ac = 0 \rightarrow$ two equal real roots

(C) If $b^2 - 4ac < 0 \rightarrow$ no real roots

(I) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -3, c = 5$

Discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40$

$= -31$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for the given equation.

(II) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 3,\ b = -4\sqrt{3}, c = 4$

Discriminant $= b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4)$

$= 48 - 48 = 0$

As $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$.

$$\frac{-b}{2a} = \frac{-\left(-4\sqrt{3}\right)}{2\times 3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(III) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, $b = -6$, $c = 3$

Discriminant $= b^2 - 4ac = (-6)^2 - 4(2)(3)$

$= 36 - 24 = 12$

As $b^2 - 4ac > 0$,

Therefore, distinct real roots exist for this equation as follows.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

Question 2:

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(I) $2x^2 + kx + 3 = 0$

(II) $kx(x - 2) + 6 = 0$

Answer:

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant

$(b^2 - 4ac)$ will be 0.

(I) $2x^2 + kx + 3 = 0$

Comparing equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, $b = k$, $c = 3$

Discriminant $= b^2 - 4ac = (k)^2 - 4(2)(3)$

$= k^2 - 24$

For equal roots,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(II) $kx\,(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = k$, $b = -2k$, $c = 6$

Discriminant $= b^2 - 4ac = (-\,2k)^2 - 4\,(k)\,(6)$

$= 4k^2 - 24k$

For equal roots,

$b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k\,(k - 6) = 0$

Either $4k = 0$ or $k = 6 = 0$

$k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

Question 3:

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2?

If so, find its length and breadth.

Answer:

Length of mango grove will be $2l$.

Area of mango grove = $(2l)(l)$

$= 2l^2$

$$2l^2 = 800$$

$$l^2 = \frac{800}{2} = 400$$

$$l^2 - 400 = 0$$

Comparing this equation with $al^2 + bl + c = 0$, we obtain

$a = 1$ $b = 0$, $c = 400$

Discriminant = $b^2 - 4ac = (0)^2 - 4 \times (1) \times (-400) = 1600$

Here, $b^2 - 4ac > 0$

Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.

$$l = \pm 20$$

However, length cannot be negative.

Therefore, breadth of mango grove = 20 m

Length of mango grove = $2 \times 20 = 40$ m

Question 4:

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer:

Let the age of one friend be x years.

Age of the other friend will be $(20 - x)$ years.

4 years ago, age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4)$

$= (16 - x)$ years

Given that,

$(x - 4)(16 - x) = 48$

$16x - 64 - x^2 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 1, b = -20, c = 112$

Discriminant $= b^2 - 4ac = (-20)^2 - 4(1)(112)$

$= 400 - 448 = -48$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for this equation and hence, this situation is not possible.

Question 5:

Is it possible to design a rectangular park of perimeter 80 and area 400 m^2? If so find its length and breadth.

Answer:

Let the length and breadth of the park be l and b.

Perimeter $= 2(l + b) = 80$

$l + b = 40$

Or, $b = 40 - l$

Area $= l \times b = l(40 - l) = 40l - l^2$

$40l - l^2 = 400$

$l^2 - 40l + 400 = 0$

Comparing this equation with

$al^2 + bl + c = 0$, we obtain

$a = 1$, $b = -40$, $c = 400$

Discriminate = $b^2 - 4ac = (-40)^2 - 4(1)(400)$

$= 1600 - 1600 = 0$

As $b^2 - 4ac = 0$,

Therefore, this equation has equal real roots. And hence, this situation is possible.

Root of this equation,

$$l = -\frac{b}{2a}$$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

Therefore, length of park, $l = 20$ m

And breadth of park, $b = 40 - l = 40 - 20 = 20$ m

Exercise 5.1

Question 1:

In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv)The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Answer:

(i) It can be observed that

Taxi fare for 1st km = 15

Taxi fare for first 2 km = 15 + 8 = 23

Taxi fare for first 3 km = 23 + 8 = 31

Taxi fare for first 4 km = 31 + 8 = 39

Clearly 15, 23, 31, 39 … forms an A.P. because every term is 8 more than the preceding term.

(ii) Let the initial volume of air in a cylinder be *V* lit. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time.

In other words, after every stroke, only $1-\frac{1}{4}=\frac{3}{4}$th part of air will remain.

Therefore, volumes will be $V,\left(\frac{3V}{4}\right),\left(\frac{3}{4}V\right)^2,\left(\frac{3}{4}V\right)^3 \ldots$

Clearly, it can be observed that the adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

(iii) Cost of digging for first metre = 150

Cost of digging for first 2 metres = 150 + 50 = 200

Cost of digging for first 3 metres = 200 + 50 = 250

Cost of digging for first 4 metres = 250 + 50 = 300

Clearly, 150, 200, 250, 300 … forms an A.P. because every term is 50 more than the preceding term.

(iv) We know that if Rs P is deposited at *r*% compound interest per annum for *n* years, our money will be $P\left(1+\frac{r}{100}\right)^n$ after *n* years.

Therefore, after every year, our money will be

$$10000\left(1+\frac{8}{100}\right),10000\left(1+\frac{8}{100}\right)^2,10000\left(1+\frac{8}{100}\right)^3,10000\left(1+\frac{8}{100}\right)^4,\ldots$$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Question 2:

Write first four terms of the A.P. when the first term *a* and the

(i) $a = 10$, $d = 10$

(ii) $a = -2$, $d = 0$

(iii) $a = 4$, $d = -3$

(iv) $a = -1$ $d = \frac{1}{2}$

(v) $a = -1.25$, $d = -0.25$

Answer:

(i) $a = 10$, $d = 10$

Let the series be a_1, a_2, a_3, a_4, a_5 …

$a_1 = a = 10$

$a_2 = a_1 + d = 10 + 10 = 20$

$a_3 = a_2 + d = 20 + 10 = 30$

$a_4 = a_3 + d = 30 + 10 = 40$

$a_5 = a_4 + d = 40 + 10 = 50$

Therefore, the series will be 10, 20, 30, 40, 50 …

First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a = -2$, $d = 0$

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = -2$

$a_2 = a_1 + d = -2 + 0 = -2$

$a_3 = a_2 + d = -2 + 0 = -2$

$a_4 = a_3 + d = -2 + 0 = -2$

Therefore, the series will be −2, −2, −2, −2 …

First four terms of this A.P. will be −2, −2, −2 and −2.

(iii) $a = 4$, $d = -3$

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = 4$

$a_2 = a_1 + d = 4 - 3 = 1$

$a_3 = a_2 + d = 1 - 3 = -2$

$a_4 = a_3 + d = -2 - 3 = -5$

Therefore, the series will be 4, 1, −2 −5 …

First four terms of this A.P. will be 4, 1, −2 and −5.

(iv) $a = -1$, $d = \frac{1}{2}$

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = -1$

$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$

$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Clearly, the series will be

$-1, -\frac{1}{2}, 0, \frac{1}{2}$ ………….

First four terms of this A.P. will be $-1, -\frac{1}{2}, 0 \text{ and } \frac{1}{2}$.

(v) $a = -1.25$, $d = -0.25$

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = -1.25$

$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$a_4 = a_3 + d = -1.75 - 0.25 = -2.00$

Clearly, the series will be 1.25, −1.50, −1.75, −2.00 ……..

First four terms of this A.P. will be −1.25, −1.50, −1.75 and −2.00.

Question 3:

For the following A.P.s, write the first term and the common difference.

(i) 3, 1, − 1, − 3 …

(ii) − 5, − 1, 3, 7 …

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

(iv) 0.6, 1.7, 2.8, 3.9 …

Answer:

(i) 3, 1, −1, −3 …

Here, first term, $a = 3$

Common difference, d = Second term − First term

$= 1 - 3 = -2$

(ii) −5, −1, 3, 7 …

Here, first term, $a = -5$

Common difference, d = Second term − First term

$= (-1) - (-5) = -1 + 5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} \ldots$

Here, first term, $a = \frac{1}{3}$

Common difference, d = Second term − First term

$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9 …

Here, first term, $a = 0.6$

Common difference, d = Second term − First term

$= 1.7 - 0.6$

$= 1.1$

Exercise 5.2

Question 1:

Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

	a	d	n	a_n
I	7	3	8	……
II	− 18	…..	10	0
III	…..	− 3	18	− 5
IV	− 18.9	2.5	…..	3.6
V	3.5	0	105	…..

Answer:

I. $a = 7$, $d = 3$, $n = 8$, $a_n = ?$

We know that,

For an A.P. $a_n = a + (n - 1) d$

$= 7 + (8 - 1) 3$

$= 7 + (7) 3$

$= 7 + 21 = 28$

Hence, $a_n = 28$

II. Given that

$a = -18$, $n = 10$, $a_n = 0$, $d = ?$

We know that,

$a_n = a + (n - 1)\,d$

$0 = -18 + (10 - 1)\,d$

$18 = 9d$

$d = \frac{18}{9} = 2$

Hence, common difference, $d = 2$

III. Given that

$d = -3$, $n = 18$, $a_n = -5$

We know that,

$a_n = a + (n - 1)\,d$

$-5 = a + (18 - 1)(-3)$

$-5 = a + (17)(-3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

Hence, $a = 46$

IV. $a = -18.9$, $d = 2.5$, $a_n = 3.6$, $n = ?$

We know that,

$a_n = a + (n - 1)\,d$

$3.6 = -18.9 + (n - 1)\,2.5$

$3.6 + 18.9 = (n - 1)\,2.5$

$22.5 = (n - 1)\,2.5$

$(n-1) = \frac{22.5}{2.5}$

$n - 1 = 9$

$n = 10$

Hence, $n = 10$

V. $a = 3.5$, $d = 0$, $n = 105$, $a_n = ?$

We know that,

$a_n = a + (n - 1)\, d$

$a_n = 3.5 + (105 - 1)\, 0$

$a_n = 3.5 + 104 \times 0$

$a_n = 3.5$

Hence, $a_n = 3.5$

Question 2:

Choose the correct choice in the following and justify

I. 30^{th} term of the A.P: 10, 7, 4, …, is

A. 97 **B.** 77 **C.** − 77 **D.** − 87

II 11^{th} term of the A.P. $-3, -\frac{1}{2}, 2, \ldots$ is

A. 28 **B.** 22 **C.** − 38 **D.** $-48\frac{1}{2}$

Answer:

I. Given that

A.P. 10, 7, 4, …

First term, $a = 10$

Common difference, $d = a_2 - a_1 = 7 - 10$

$= -3$

We know that, $a_n = a + (n - 1)\, d$

$a_{30} = 10 + (30 - 1)(-3)$

$a_{30} = 10 + (29)(-3)$

$a_{30} = 10 - 87 = -77$

Hence, the correct answer is **C**.

II. Given that, A.P. $-3, -\frac{1}{2}, 2, \ldots$

First term $a = -3$

Common difference, $d = a_2 - a_1$

$$= -\frac{1}{2} - (-3)$$
$$= -\frac{1}{2} + 3 = \frac{5}{2}$$

We know that,

$$a_n = a + (n-1)d$$
$$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$$
$$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$$
$$a_{11} = -3 + 25$$
$$a_{11} = 22$$

Hence, the answer is **B**.

Question 3:

In the following APs find the missing term in the boxes

I. $2, \square, 26$

II. $\square, 13, \square, 3$

III. $5, \square, \square, 9\frac{1}{2}$

IV. $-4, \square, \square, \square, \square, 6$

v. $\square, 38, \square, \square, \square, -22$

Answer:

I. $2, \square, 26$

For this A.P.,

$a = 2$

$a_3 = 26$

We know that, $a_n = a + (n - 1) d$

$a_3 = 2 + (3 - 1) d$

$26 = 2 + 2d$

$24 = 2d$

$d = 12$

$a_2 = 2 + (2 - 1) 12$

$= 14$

Therefore, 14 is the missing term.

II. $\square, 13, \square, 3$

For this A.P.,

$a_2 = 13$ and

$a_4 = 3$

We know that, $a_n = a + (n - 1) d$

$a_2 = a + (2 - 1) d$

$13 = a + d$ (I)

$a_4 = a + (4 - 1) d$

$3 = a + 3d$ (II)

On subtracting (I) from (II), we obtain

$-10 = 2d$

$d = -5$

From equation (I), we obtain

$13 = a + (-5)$

$a = 18$

$a_3 = 18 + (3 - 1)(-5)$

$= 18 + 2(-5) = 18 - 10 = 8$

Therefore, the missing terms are 18 and 8 respectively.

III. $5, \square, \square, 9\frac{1}{2}$

For this A.P.,

$a = 5$

$a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know that,

$$a_n = a + (n-1)d$$
$$a_4 = a + (4-1)d$$
$$\frac{19}{2} = 5 + 3d$$
$$\frac{19}{2} - 5 = 3d$$
$$\frac{9}{2} = 3d$$
$$d = \frac{3}{2}$$
$$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$$
$$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

IV. $-4, \square, \square, \square, \square, 6$

For this A.P.,

$a = -4$ and

$a_6 = 6$

We know that,

$a_n = a + (n - 1)\, d$

$a_6 = a + (6 - 1)\, d$

$6 = -4 + 5d$

$10 = 5d$

$d = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2\ (2) = 0$

$a_4 = a + 3d = -4 + 3\ (2) = 2$

$a_5 = a + 4d = -4 + 4\ (2) = 4$

Therefore, the missing terms are −2, 0, 2, and 4 respectively.

v. $\square, 38, \square, \square, \square, -22$

For this A.P.,

$a_2 = 38$

$a_6 = -22$

We know that

$a_n = a + (n - 1)\ d$

$a_2 = a + (2 - 1)\ d$

$38 = a + d$ (1)

$a_6 = a + (6 - 1)\ d$

$-22 = a + 5d$ (2)

On subtracting equation (1) from (2), we obtain

$-22 - 38 = 4d$

$-60 = 4d$

$d = -15$

$a = a_2 - d = 38 - (-15) = 53$

$a_3 = a + 2d = 53 + 2\ (-15) = 23$

$a_4 = a + 3d = 53 + 3\ (-15) = 8$

$a_5 = a + 4d = 53 + 4\ (-15) = -7$

Therefore, the missing terms are 53, 23, 8, and −7 respectively.

Question 4:

Which term of the A.P. 3, 8, 13, 18, … is 78?

Answer:

3, 8, 13, 18, …

For this A.P.,

$a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this A.P. be 78.

$a_n = a + (n - 1)\, d$

$78 = 3 + (n - 1)\, 5$

$75 = (n - 1)\, 5$

$(n - 1) = 15$

$n = 16$

Hence, 16^{th} term of this A.P. is 78.

Question 5:

Find the number of terms in each of the following A.P.

I. 7, 13, 19, …, 205

II. $18, 15\frac{1}{2}, 13, \ldots, -47$

Answer:

I. 7, 13, 19, …, 205

For this A.P.,

$a = 7$

$d = a_2 - a_1 = 13 - 7 = 6$

Let there are n terms in this A.P.

$a_n = 205$

We know that

$a_n = a + (n - 1)\, d$

Therefore, $205 = 7 + (n - 1)\, 6$

$198 = (n - 1)\, 6$

$33 = (n - 1)$

$n = 34$

Therefore, this given series has 34 terms in it.

II. $18, 15\frac{1}{2}, 13, \ldots, -47$

For this A.P.,

$a = 18$

$d = a_2 - a_1 = 15\frac{1}{2} - 18$

$d = \frac{31 - 36}{2} = -\frac{5}{2}$

Let there are n terms in this A.P.

Therefore, $a_n = -47$ and we know that,

$$a_n = a + (n-1)d$$
$$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$$
$$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$$
$$-65 = (n-1)\left(-\frac{5}{2}\right)$$
$$(n-1) = \frac{-130}{-5}$$
$$(n-1) = 26$$
$$n = 27$$

Therefore, this given A.P. has 27 terms in it.

Question 6:

Check whether – 150 is a term of the A.P. 11, 8, 5, 2, …

Answer:

For this A.P.,

a = 11

$d = a_2 - a_1 = 8 - 11 = -3$

Let −150 be the n^{th} term of this A.P.

We know that,

$$a_n = a + (n-1)d$$
$$-150 = 11 + (n-1)(-3)$$
$$-150 = 11 - 3n + 3$$
$$-164 = -3n$$
$$n = \frac{164}{3}$$

Clearly, n is not an integer.

Therefore, −150 is not a term of this A.P.

Question 7:

Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73

Answer:

Given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n - 1) d$

$a_{11} = a + (11 - 1) d$

$38 = a + 10d$ (1)

Similarly,

$a_{16} = a + (16 - 1) d$

$73 = a + 15d$ (2)

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

$38 = a + 10 \times (7)$

$38 - 70 = a$

$a = -32$

$a_{31} = a + (31 - 1) d$

$= -32 + 30 (7)$

$= -32 + 210$

$= 178$

Hence, 31st term is 178.

Question 8:

An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term

Answer:

Given that,

$a_3 = 12$

$a_{50} = 106$

We know that,

$a_n = a + (n - 1)\ d$

$a_3 = a + (3 - 1)\ d$

$12 = a + 2d$ (I)

Similarly, $a_{50} = a + (50 - 1)\ d$

$106 = a + 49d$ (II)

On subtracting (I) from (II), we obtain

$94 = 47d$

$d = 2$

From equation (I), we obtain

$12 = a + 2\ (2)$

$a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)\ d$

$a_{29} = 8 + (28)2$

$a_{29} = 8 + 56 = 64$

Therefore, 29th term is 64

Question 9:

If the 3rd and the 9th terms of an A.P. are 4 and − 8 respectively.

Which term of this A.P. is zero.

Answer:

Given that,

$a_3 = 4$

$a_9 = -8$

We know that,

$a_n = a + (n - 1)\,d$

$a_3 = a + (3 - 1)\,d$

$4 = a + 2d$ (I)

$a_9 = a + (9 - 1)\,d$

$-8 = a + 8d$ (II)

On subtracting equation (I) from (II), we obtain

$-12 = 6d$

$d = -2$

From equation (I), we obtain

$4 = a + 2\,(-2)$

$4 = a - 4$

$a = 8$

Let n^{th} term of this A.P. be zero.

$a_n = a + (n - 1)\,d$

$0 = 8 + (n - 1)\,(-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5^{th} term of this A.P. is 0.

Question 10:

If 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Answer:

We know that,

For an A.P., $a_n = a + (n - 1)\, d$

$a_{17} = a + (17 - 1)\, d$

$a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$

It is given that

$a_{17} - a_{10} = 7$

$(a + 16d) - (a + 9d) = 7$

$7d = 7$

$d = 1$

Therefore, the common difference is 1.

Question 11:

Which term of the A.P. 3, 15, 27, 39, … will be 132 more than its 54^{th} term?

Answer:

Given A.P. is 3, 15, 27, 39, …

$a = 3$

$d = a_2 - a_1 = 15 - 3 = 12$

$a_{54} = a + (54 - 1)\, d$

$= 3 + (53)(12)$

$= 3 + 636 = 639$

$132 + 639 = 771$

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.

$a_n = a + (n - 1) d$

$771 = 3 + (n - 1) 12$

$768 = (n - 1) 12$

$(n - 1) = 64$

$n = 65$

Therefore, 65^{th} term was 132 more than 54^{th} term.

Alternatively,

Let n^{th} term be 132 more than 54^{th} term.

$$n = 54 + \frac{132}{12}$$

$$= 54 + 11 = 65^{th} \text{ term}$$

Question 12:

Two APs have the same common difference. The difference between their 100^{th} term is 100, what is the difference between their 1000^{th} terms?

Answer:

Let the first term of these A.P.s be a_1 and a_2 respectively and the common difference of these A.P.s be d.

For first A.P.,

$a_{100} = a_1 + (100 - 1) d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)\ d$

$a_{1000} = a_1 + 999d$

For second A.P.,

$a_{100} = a_2 + (100 - 1)\ d$

$= a_2 + 99d$

$a_{1000} = a_2 + (1000 - 1)\ d$

$= a_2 + 999d$

Given that, difference between

100th term of these A.P.s = 100

Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$

$a_1 - a_2 = 100$ (1)

Difference between 1000th terms of these A.P.s

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1),

This difference, $a_1 - a_2 = 100$

Hence, the difference between 1000th terms of these A.P. will be 100.

Question 13:

How many three digit numbers are divisible by 7

Answer:

First three-digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore, 105, 112, 119, …

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5. Clearly, 999 − 5 = 994 is the maximum possible three-digit number that is divisible by 7.

The series is as follows.

105, 112, 119, …, 994

Let 994 be the nth term of this A.P.

$a = 105$

$d = 7$

$a_n = 994$

$n = ?$

$a_n = a + (n - 1)\, d$

$994 = 105 + (n - 1)\, 7$

$889 = (n - 1)\, 7$

$(n - 1) = 127$

$n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

Question 14:

How many multiples of 4 lie between 10 and 250?

Answer:

First multiple of 4 that is greater than 10 is 12. Next will be 16.

Therefore, 12, 16, 20, 24, …

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2. Therefore, 250 − 2 = 248 is divisible by 4.

The series is as follows.

12, 16, 20, 24, …, 248

Let 248 be the n^{th} term of this A.P.

$$a = 12$$
$$d = 4$$
$$a_n = 248$$
$$a_n = a + (n-1)d$$
$$248 = 12 + (n-1)4$$
$$\frac{236}{4} = n - 1$$
$$59 = n - 1$$
$$n = 60$$

Therefore, there are 60 multiples of 4 between 10 and 250.

Question 15:

For what value of n, are the n^{th} terms of two APs 63, 65, 67, and 3, 10, 17, … equal

Answer:

63, 65, 67, …

$a = 63$

$d = a_2 - a_1 = 65 - 63 = 2$

n^{th} term of this A.P. $= a_n = a + (n - 1)\, d$

$a_n = 63 + (n - 1)\, 2 = 63 + 2n - 2$

$a_n = 61 + 2n$ (1)

3, 10, 17, …

$a = 3$

$d = a_2 - a_1 = 10 - 3 = 7$

n^{th} term of this A.P. = $3 + (n - 1)\ 7$

$a_n = 3 + 7n - 7$

$a_n = 7n - 4$ (2)

It is given that, n^{th} term of these A.P.s are equal to each other.

Equating both these equations, we obtain

$61 + 2n = 7n - 4$

$61 + 4 = 5n$

$5n = 65$

$n = 13$

Therefore, 13^{th} terms of both these A.P.s are equal to each other.

Question 16:

Determine the A.P. whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Answer:

$=a_3 = 16$

$a + (3 - 1)\ d = 16$

$a + 2d = 16$ (1)

$a_7 - a_5 = 12$

$[a + (7 - 1)\ d] - [a + (5 - 1)\ d] = 12$

$(a + 6d) - (a + 4d) = 12$

$2d = 12$

$d = 6$

From equation (1), we obtain

$a + 2(6) = 16$

$a + 12 = 16$

$a = 4$

Therefore, A.P. will be

4, 10, 16, 22, …

Question 17:

Find the 20^{th} term from the last term of the A.P. 3, 8, 13, …, 253

Answer:

Given A.P. is

3, 8, 13, …, 253

Common difference for this A.P. is 5.

Therefore, this A.P. can be written in reverse order as

253, 248, 243, …, 13, 8, 5

For this A.P.,

$a = 253$

$d = 248 - 253 = -5$

$n = 20$

$a_{20} = a + (20 - 1)\, d$

$a_{20} = 253 + (19)(-5)$

$a_{20} = 253 - 95$

$a = 158$

Therefore, 20^{th} term from the last term is 158.

Question 18:

The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the A.P.

Answer:

We know that,

$a_n = a + (n - 1)d$

$a_4 = a + (4 - 1)d$

$a_4 = a + 3d$

Similarly,

$a_8 = a + 7d$

$a_6 = a + 5d$

$a_{10} = a + 9d$

Given that, $a_4 + a_8 = 24$

$a + 3d + a + 7d = 24$

$2a + 10d = 24$

$a + 5d = 12$ (1)

$a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$

$a + 7d = 22$ (2)

On subtracting equation (1) from (2), we obtain

$2d = 22 - 12$

$2d = 10$

$d = 5$

From equation (1), we obtain

$a + 5d = 12$

$a + 5\ (5) = 12$

$a + 25 = 12$

$a = -13$

$a_2 = a + d = -13 + 5 = -8$

$a_3 = a_2 + d = -8 + 5 = -3$

Therefore, the first three terms of this A.P. are −13, −8, and −3.

Question 19:

Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Answer:

It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his salary is increased by Rs 200.

Therefore, the salaries of each year after 1995 are

5000, 5200, 5400, …

Here, $a = 5000$

$d = 200$

Let after n^{th} year, his salary be Rs 7000.

Therefore, $a_n = a + (n - 1)\ d$

$7000 = 5000 + (n - 1)\ 200$

$200(n - 1) = 2000$

$(n - 1) = 10$

$n = 11$

Therefore, in 11th year, his salary will be Rs 7000.

Question 20:

Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the n^{th} week, her week, her weekly savings become Rs 20.75, find *n.*

Answer:

Given that,

$a = 5$

$d = 1.75$

$a_n = 20.75$

$n = ?$

$a_n = a + (n - 1)\, d$

$$20.75 = 5 + (n-1)1.75$$

$$15.75 = (n-1)1.75$$

$$(n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$$

$$= \frac{63}{7} = 9$$

$n - 1 = 9$

$n = 10$

Hence, n is 10.

Exercise 5.3

Question 1:

Find the sum of the following APs.

(i) 2, 7, 12 ,…., to 10 terms.

(ii) − 37, − 33, − 29 ,…, to 12 terms

(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,………, to 11 terms

Answer:

(i)2, 7, 12 ,…, to 10 terms

For this A.P.,

$a = 2$

$d = a_2 - a_1 = 7 - 2 = 5$

$n = 10$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(2) + (10-1)5\right]$$

$$= 5\left[4 + (9)\times(5)\right]$$

$$= 5 \times 49 = 245$$

(ii)−37, −33, −29 ,…, to 12 terms

For this A.P.,

$a = -37$

$d = a_2 - a_1 = (-33) - (-37)$

$= -33 + 37 = 4$

n = 12

We know that,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(-37)+(12-1)4\right]$$

$$= 6\left[-74+11\times 4\right]$$

$$= 6\left[-74+44\right]$$

$$= 6(-30) = -180$$

(iii) 0.6, 1.7, 2.8 ,…, to 100 terms

For this A.P.,

a = 0.6

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

n = 100

We know that,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2(0.6)+(100-1)1.1\right]$$

$$= 50\left[1.2+(99)\times(1.1)\right]$$

$$= 50\left[1.2+108.9\right]$$

$$= 50\left[110.1\right]$$

$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10},$. …….. , to 11 terms

For this A.P.,

$a = \frac{1}{15}$

n = 11

$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15}$

$= \frac{5-4}{60} = \frac{1}{60}$

We know that,

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$

$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$

Question 2:

Find the sums given below

(i) 7 + $10\frac{1}{2}$ + 14 + ………… + 84

(ii) 34 + 32 + 30 + ……….. + 10

(iii) − 5 + (− 8) + (− 11) + ………… + (− 230)

Answer:

(i)7 + $10\frac{1}{2}$ + 14 + …………+ 84

For this A.P.

$a = 7$

$l = 84$

$$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$$

Let 84 be the n^{th} term of this A.P.

$l = a + (n - 1)d$

$$84 = 7 + (n-1)\frac{7}{2}$$

$$77 = (n-1)\frac{7}{2}$$

$22 = n - 1$

$n = 23$

We know that,

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{23}{2}[7+84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii)34 + 32 + 30 + ……….. + 10

For this A.P.,

$a = 34$

$d = a_2 - a_1 = 32 - 34 = -2$

$l = 10$

Let 10 be the n^{th} term of this A.P.

$l = a + (n - 1)\ d$

$10 = 34 + (n - 1)\ (-2)$

$-24 = (n - 1)\ (-2)$

$12 = n - 1$

$n = 13$

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{13}{2}(34+10)$$

$$= \frac{13\times 44}{2} = 13\times 22$$

$$= 286$$

(iii)$(-5) + (-8) + (-11) + \ldots\ldots\ldots\ldots + (-230)$

For this A.P.,

$a = -5$

$l = -230$

$d = a_2 - a_1 = (-8) - (-5)$

$= - 8 + 5 = -3$

Let -230 be the n^{th} term of this A.P.

$l = a + (n - 1)d$

$-230 = - 5 + (n - 1)\ (-3)$

$-225 = (n - 1)\ (-3)$

$(n - 1) = 75$

$n = 76$

And, $S_n = \frac{n}{2}(a+l)$

$= \frac{76}{2}\left[(-5)+(-230)\right]$

$= 38(-235)$

$= -8930$

Question 3:

In an AP

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

(x)Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.

Answer:

(i) Given that, $a = 5$, $d = 3$, $a_n = 50$

As $a_n = a + (n - 1)d$,

$\therefore 50 = 5 + (n - 1)3$

$45 = (n - 1)3$

$15 = n - 1$

$n = 16$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{16} = \frac{16}{2}[5 + 50]$$

$$= 8 \times 55$$

$$= 440$$

(ii) Given that, $a = 7$, $a_{13} = 35$

As $a_n = a + (n - 1) d$,

$\therefore a_{13} = a + (13 - 1) d$

$35 = 7 + 12 d$

$35 - 7 = 12d$

$28 = 12d$

$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{13} = \frac{n}{2}[a + a_{13}]$$

$$= \frac{13}{2}[7 + 35]$$

$$= \frac{13 \times 42}{2} = 13 \times 21$$

$$= 273$$

(iii)Given that, $a_{12} = 37$, $d = 3$

As $a_n = a + (n - 1)d$,

$a_{12} = a + (12 - 1)3$

$37 = a + 33$

$a = 4$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_n = \frac{12}{2}[4 + 37]$$

$$S_n = 6(41)$$

$$S_n = 246$$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

As $a_n = a + (n - 1)d$,

$a_3 = a + (3 - 1)d$

$15 = a + 2d$ (i)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2a + (10-1)d]$$

$$125 = 5(2a + 9d)$$

$$25 = 2a + 9d \qquad \text{(ii)}$$

On multiplying equation (1) by 2, we obtain

$30 = 2a + 4d$ (iii)

On subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (i),

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10 - 1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v)Given that, $d = 5$, $S_9 = 75$

As $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$,

$S_9 = \frac{9}{2}\left[2a + (9-1)5\right]$

$75 = \frac{9}{2}(2a + 40)$

$25 = 3(a + 20)$

$25 = 3a + 60$

$3a = 25 - 60$

$a = \frac{-35}{3}$

$a_n = a + (n - 1)d$

$a_9 = a + (9 - 1)(5)$

$= \frac{-35}{3} + 8(5)$

$= \frac{-35}{3} + 40$

$= \frac{-35+120}{3} = \frac{85}{3}$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

As $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$,

$90 = \frac{n}{2}\left[4 + (n-1)8\right]$

$90 = n\,[2 + (n - 1)4]$

$90 = n\,[2 + 4n - 4]$

$90 = n(4n - 2) = 4n^2 - 2n$

$4n^2 - 2n - 90 = 0$

$4n^2 - 20n + 18n - 90 = 0$

$4n(n - 5) + 18(n - 5) = 0$

$(n - 5)(4n + 18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$n = 5$ or $n = -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

$a_n = a + (n - 1)d$

$a_5 = 2 + (5 - 1)8$

$= 2 + (4)(8)$

$= 2 + 32 = 34$

(vii) Given that, $a = 8$, $a_n = 62$, $S_n = 210$

$S_n = \frac{n}{2}[a + a_n]$

$210 = \frac{n}{2}[8 + 62]$

$210 = \frac{n}{2}(70)$

$n = 6$

$a_n = a + (n - 1)d$

$62 = 8 + (6 - 1)d$

$62 - 8 = 5d$

$54 = 5d$

$d = \frac{54}{5}$

(viii) Given that, $a_n = 4$, $d = 2$, $S_n = -14$

$a_n = a + (n - 1)d$

$4 = a + (n - 1)2$

$4 = a + 2n - 2$

$a + 2n = 6$

$a = 6 - 2n$ (i)

$S_n = \frac{n}{2}[a + a_n]$

$-14 = \frac{n}{2}[a + 4]$

$-28 = n\ (a + 4)$

$-28 = n\ (6 - 2n + 4)$ {From equation (i)}

$-28 = n\ (-\ 2n + 10)$

$-28 = -\ 2n^2 + 10n$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n\ (n - 7) + 2(n - 7) = 0$

$(n - 7)\ (n + 2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$

$n = 7$ or $n = -2$

However, n can neither be negative nor fractional.

Therefore, $n = 7$

From equation (i), we obtain

$a = 6 - 2n$

$a = 6 - 2(7)$

$= 6 - 14$

$= -8$

(ix)Given that, $a = 3$, $n = 8$, $S = 192$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$192 = \frac{8}{2}\left[2\times 3 + (8-1)d\right]$$

$192 = 4\ [6 + 7d]$

$48 = 6 + 7d$

$42 = 7d$

$d = 6$

(x)Given that, $l = 28$, $S = 144$ and there are total of 9 terms.

$$S_n = \frac{n}{2}(a + l)$$

$$144 = \frac{9}{2}(a + 28)$$

$(16) \times (2) = a + 28$

$32 = a + 28$

$a = 4$

Question 4:

How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Answer:

Let there be n terms of this A.P.

For this A.P., $a = 9$

$d = a_2 - a_1 = 17 - 9 = 8$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$636 = \frac{n}{2}\left[2 \times a + (n-1)8\right]$$

$$636 = \frac{n}{2}\left[18 + (n-1)8\right]$$

$636 = n\,[9 + 4n - 4]$

$636 = n\,(4n + 5)$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n\,(4n + 53) - 12\,(4n + 53) = 0$

$(4n + 53)\,(n - 12) = 0$

Either $4n + 53 = 0$ or $n - 12 = 0$

$n = \frac{-53}{4}$ or $n = 12$

n cannot be $-\frac{53}{4}$. As the number of terms can neither be negative nor fractional, therefore, $n = 12$ only.

Question 5:

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer:

Given that,

$a = 5$

$l = 45$

$S_n = 400$

$$S_n = \frac{n}{2}(a+l)$$

$$400 = \frac{n}{2}(5+45)$$

$$400 = \frac{n}{2}(50)$$

$n = 16$

$l = a + (n - 1)\ d$

$45 = 5 + (16 - 1)\ d$

$40 = 15d$

$$d = \frac{40}{15} = \frac{8}{3}$$

Question 6:

The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer:

Given that,

$a = 17$

$l = 350$

$d = 9$

Let there be n terms in the A.P.

$l = a + (n - 1)\ d$

$350 = 17 + (n - 1)9$

$333 = (n - 1)9$

$(n - 1) = 37$

$n = 38$

$$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Question 7:

Find the sum of first 22 terms of an AP in which $d = 7$ and 22^{nd} term is 149.

Answer:

$d = 7$

$a_{22} = 149$

$S_{22} = ?$

$a_n = a + (n - 1)d$

$a_{22} = a + (22 - 1)d$

$149 = a + 21 \times 7$

$149 = a + 147$

$a = 2$

$$S_n = \frac{n}{2}(a + a_n)$$

$$= \frac{22}{2}(2 + 149)$$

$$= 11(151) = 1661$$

Question 8:

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer:

Given that,

$a_2 = 14$

$a_3 = 18$

$d = a_3 - a_2 = 18 - 14 = 4$

$a_2 = a + d$

$14 = a + 4$

$a = 10$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{51} = \frac{51}{2}\left[2 \times 10 + (51-1)4\right]$$

$$= \frac{51}{2}\left[20 + (50)(4)\right]$$

$$= \frac{51(220)}{2} = 51(110)$$

$= 5610$

Question 9:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer:

Given that,

$S_7 = 49$

$S_{17} = 289$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_7 = \frac{7}{2}\left[2a + (7-1)d\right]$$

$$49 = \frac{7}{2}(2a + 6d)$$

$7 = (a + 3d)$

$a + 3d = 7$ (i)

Similarly, $$S_{17} = \frac{17}{2}\left[2a + (17-1)d\right]$$

$$289 = \frac{17}{2}\left[2a + 16d\right]$$

$17 = (a + 8d)$

$a + 8d = 17$ (ii)

Subtracting equation (i) from equation (ii),

$5d = 10$

$d = 2$

From equation (i),

$a + 3(2) = 7$

$a + 6 = 7$

$a = 1$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$= \frac{n}{2}\left[2(1)+(n-1)(2)\right]$$
$$= \frac{n}{2}(2+2n-2)$$
$$= \frac{n}{2}(2n)$$

$= n^2$

Question 10:

Show that a_1, a_2 … , a_n , … form an AP where a_n is defined as below

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Answer:

(i) $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

It can be observed that

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(7)+(15-1)4\right]$$

$$= \frac{15}{2}\left[(14)+56\right]$$

$$= \frac{15}{2}(70)$$

= 15 × 35

= 525

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

It can be observed that

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 - (-1) = -5$

$a_4 - a_3 = -11 - (-6) = -5$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as −5 and first term as 4.

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(4)+(15-1)(-5)\right]$$

$$= \frac{15}{2}\left[8+14(-5)\right]$$

$$= \frac{15}{2}(8-70)$$

$$= \frac{15}{2}(-62) = 15(-31)$$

$= -465$

Question 11:

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the10th and the nth terms.

Answer:

Given that,

$S_n = 4n - n^2$

First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms $= S_2$

$= 4(2) - (2)^2 = 8 - 4 = 4$

Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

$a_n = a + (n - 1)d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$

$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4. The second term is 1. 3^{rd}, 10^{th}, and n^{th} terms are -1, -15, and $5 - 2n$ respectively.

Question 12:

Find the sum of first 40 positive integers divisible by 6.

Answer:

The positive integers that are divisible by 6 are

6, 12, 18, 24 …

It can be observed that these are making an A.P. whose first term is 6 and common difference is 6.

$a = 6$

$d = 6$

$S_{40} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2(6) + (40-1)6\right]$$

$= 20[12 + (39)(6)]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Question 13:

Find the sum of first 15 multiples of 8.

Answer:

The multiples of 8 are

8, 16, 24, 32…

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, $a = 8$

$d = 8$

$S_{15} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{15}{2}\left[2(8) + (15-1)8\right]$$

$$= \frac{15}{2}\left[16 + 14(8)\right]$$

$$= \frac{15}{2}(16 + 112)$$

$$= \frac{15(128)}{2} = 15 \times 64$$

$= 960$

Question 14:

Find the sum of the odd numbers between 0 and 50.

Answer:

The odd numbers between 0 and 50 are

1, 3, 5, 7, 9 … 49

Therefore, it can be observed that these odd numbers are in an A.P.

$a = 1$

$d = 2$

$l = 49$

$l = a + (n - 1)\ d$

$49 = 1 + (n - 1)2$

$48 = 2(n - 1)$

$n - 1 = 24$

$n = 25$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{25} = \frac{25}{2}(1 + 49)$$

$$= \frac{25(50)}{2} = (25)(25)$$

$= 625$

Question 15:

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Answer:

It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

$a = 200$

$d = 50$

Penalty that has to be paid if he has delayed the work by 30 days =

S

$= \frac{30}{2}\left[2(200)+(30-1)50\right]$

= 15 [400 + 1450]

= 15 (1850)

= 27750

Therefore, the contractor has to pay Rs 27750 as penalty.

Question 16:

A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer:

Let the cost of 1^{st} prize be P.

Cost of 2^{nd} prize = $P - 20$

And cost of 3^{rd} prize = $P - 40$

It can be observed that the cost of these prizes are in an A.P. having common difference as −20 and first term as P.

$a = P$

$d = -20$

Given that, $S_7 = 700$

$\frac{7}{2}\left[2a+(7-1)d\right]=700$

$\frac{\left[2a+(6)(-20)\right]}{2}=100$

$a + 3(-20) = 100$

$a - 60 = 100$

a = 160

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Question 17:

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Answer:

It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5………………..12

First term, a = 1

Common difference, d = 2 − 1 = 1

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(1)+(12-1)(1)\right]$$

= 6 (2 + 11)

= 6 (13)

= 78

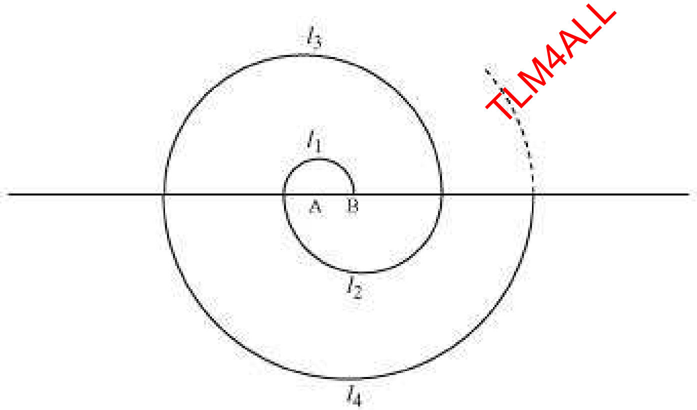
Therefore, number of trees planted by 1 section of the classes = 78

Number of trees planted by 3 sections of the classes = 3 × 78 = 234

Therefore, 234 trees will be planted by the students.

Question 18:

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? $\left(\text{Take } \pi = \frac{22}{7}\right)$



Answer:

Semi-perimeter of circle = πr

$I_1 = \pi(0.5) = \frac{\pi}{2}$ cm

$I_2 = \pi(1) = \pi$ cm

$I_3 = \pi(1.5) = \frac{3\pi}{2}$ cm

Therefore, I_1, I_2, I_3 ,i.e. the lengths of the semi-circles are in an A.P.,

$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots\ldots\ldots\ldots$

$a = \frac{\pi}{2}$

$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

$S_{13} = ?$

We know that the sum of *n* terms of an a A.P. is given by

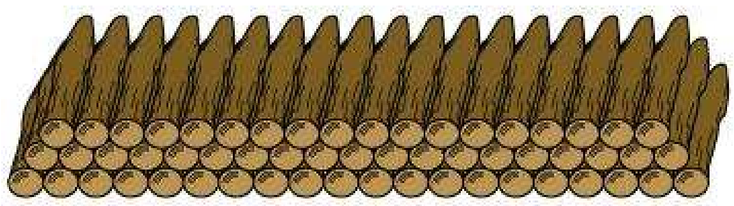
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right]$$
$$= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right]$$
$$= \left(\frac{13}{2}\right)(7\pi)$$
$$= \frac{91\pi}{2}$$
$$= \frac{91 \times 22}{2 \times 7} = 13 \times 11$$

= 143

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Question 19:

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Answer:

It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18…

For this A.P.,

$a = 20$

$d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

$S_n = 200$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$200 = \frac{n}{2}\left[2(20) + (n-1)(-1)\right]$$

$400 = n\ (40 - n + 1)$

$400 = n\ (41 - n)$

$400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n\ (n - 16) - 25\ (n - 16) = 0$

$(n - 16)\ (n - 25) = 0$

Either $(n - 16) = 0$ or $n - 25 = 0$

$n = 16$ or $n = 25$

$a_n = a + (n - 1)d$

$a_{16} = 20 + (16 - 1)(-1)$

$a_{16} = 20 - 15$

$a_{16} = 5$

Similarly,

$a_{25} = 20 + (25 - 1)(-1)$

$a_{25} = 20 - 24$

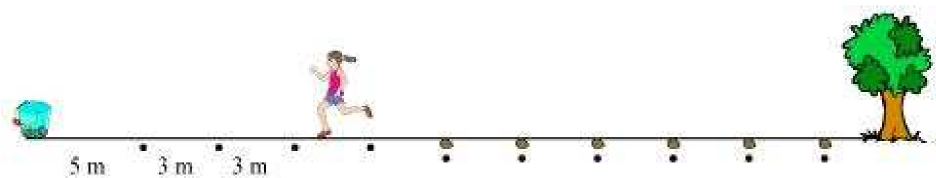
$= -4$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

Question 20:

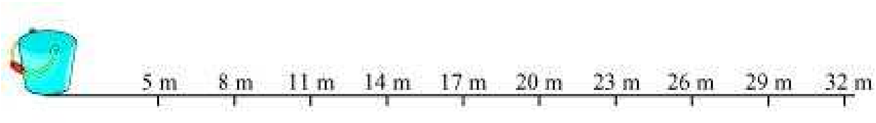
In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 × 5 + 2 ×(5 + 3)]

Answer:



The distances of potatoes are as follows.

5, 8, 11, 14…

It can be observed that these distances are in A.P.

$a = 5$

$d = 8 - 5 = 3$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(5) + (10-1)3\right]$$

= 5[10 + 9 × 3]

= 5(10 + 27) = 5(37)

= 185

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.

Therefore, total distance that the competitor will run = 2 × 185

= 370 m

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14…

Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept. Therefore, distances to be run are

10, 16, 22, 28, 34,……….

a = 10

d = 16 − 10 = 6

S_{10} =?

$$S_{10} = \frac{10}{2}\left[2\times 10+\left(10-1\right)6\right]$$

= 5[20 + 54]

= 5 (74)

= 370

Therefore, the competitor will run a total distance of 370 m.

Exercise 5.4

Question 1:

Which term of the A.P. 121, 117, 113 … is its first negative term?

[Hint: Find n for $a_n < 0$]

Answer:

Given A.P. is 121, 117, 113 …

$a = 121$

$d = 117 - 121 = -4$

$a_n = a + (n - 1) d$

$= 121 + (n - 1)(-4)$

$= 121 - 4n + 4$

$= 125 - 4n$

We have to find the first negative term of this A.P.

$\text{Therefore}, a_n < 0$

$125 - 4n < 0$

$125 < 4n$

$n > \frac{125}{4}$

$n > 31.25$

Therefore, 32^{nd} term will be the first negative term of this A.P.

Question 2:

The sum of the third and the seventh terms of an A.P is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Answer:

We know that,

$a_n = a + (n - 1)\ d$

$a_3 = a + (3 - 1)\ d$

$a_3 = a + 2d$

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$(a + 2d) + (a + 6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 - 4d$ (*i*)

Also, it is given that $(a_3) \times (a_7) = 8$

$(a + 2d) \times (a + 6d) = 8$

From equation (*i*),

$$(3-4d+2d)\times(3-4d+6d)=8$$

$$(3-2d)\times(3+2d)=8$$

$$9-4d^2=8$$

$$4d^2=9-8=1$$

$$d^2=\frac{1}{4}$$

$$d=\pm\frac{1}{2}$$

$$d=\frac{1}{2} \text{ or } -\frac{1}{2}$$

From equation (*i*),

$$\left(\text{When } d \text{ is } \frac{1}{2}\right)$$

$$a=3-4d$$

$$a = 3 - 4\left(\frac{1}{2}\right)$$

$$= 3 - 2 = 1$$

$$\left(\text{When } d \text{ is } -\frac{1}{2}\right)$$

$$a = 3 - 4\left(-\frac{1}{2}\right)$$

$$a = 3 + 2 = 5$$

$$S_n = \frac{n}{2}\left[2a(n-1)d\right]$$

$$\left(\text{When } a \text{ is } 1 \text{ and } d \text{ is } \frac{1}{2}\right)$$

$$S_{16} = \frac{16}{2}\left[2(1) + (16-1)\left(\frac{1}{2}\right)\right]$$

$$= 8\left[2 + \frac{15}{2}\right]$$

$$= 4(19) = 76$$

$$\left(\text{When } a \text{ is } 5 \text{ and } d \text{ is } -\frac{1}{2}\right)$$

$$S_{16} = \frac{16}{2}\left[2(5) + (16-1)\left(-\frac{1}{2}\right)\right]$$

$$= 8\left[10 + (15)\left(-\frac{1}{2}\right)\right]$$

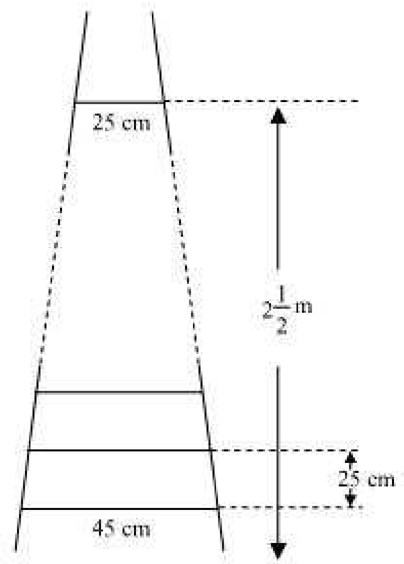
$$= 8\left(\frac{5}{2}\right)$$

$$= 20$$

Question 3:

A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[Hint: number of rungs $=\frac{250}{25}$]



Answer:

It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

$\therefore$ Total number of rungs $=\dfrac{2\frac{1}{2}\times 100}{25}+1=\dfrac{250}{25}+1=11$

Now, as the lengths of the rungs decrease uniformly, they will be in an A.P.

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

First term, $a = 45$

Last term, $l = 25$

$n = 11$

$$S_n = \frac{n}{2}(a + l)$$

$$\therefore S_{10} = \frac{11}{2}(45 + 25) = \frac{11}{2}(70) = 385 \text{ cm}$$

Therefore, the length of the wood required for the rungs is 385 cm.

Question 4:

The houses of a row are number consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it.

Find this value of x.

[Hint $S_{x-1} = S_{49} - S_x$]

Answer:

The number of houses was

1, 2, 3 … 49

It can be observed that the number of houses are in an A.P. having a as 1 and d also as 1.

Let us assume that the number of x^{th} house was like this.

We know that,

Sum of n terms in an A.P. $= \frac{n}{2}\left[2a+(n-1)d\right]$

Sum of number of houses preceding x^{th} house = S_{x-1}

$$= \frac{(x-1)}{2}\left[2a+(x-1-1)d\right]$$

$$= \frac{x-1}{2}\left[2(1)+(x-2)(1)\right]$$

$$= \frac{x-1}{2}\left[2+x-2\right]$$

$$= \frac{(x)(x-1)}{2}$$

Sum of number of houses following x^{th} house = $S_{49} - S_x$

$$= \frac{49}{2}\left[2(1)+(49-1)(1)\right] - \frac{x}{2}\left[2(1)+(x-1)(1)\right]$$

$$= \frac{49}{2}(2+49-1) - \frac{x}{2}(2+x-1)$$

$$= \left(\frac{49}{2}\right)(50) - \frac{x}{2}(x+1)$$

$$= 25(49) - \frac{x(x+1)}{2}$$

It is given that these sums are equal to each other.

$$\frac{x(x-1)}{2} = 25(49) - x\left(\frac{x+1}{2}\right)$$

$$\frac{x^2}{2} - \frac{x}{2} = 1225 - \frac{x^2}{2} - \frac{x}{2}$$

$$x^2 = 1225$$

$$x = \pm 35$$

However, the house numbers are positive integers.

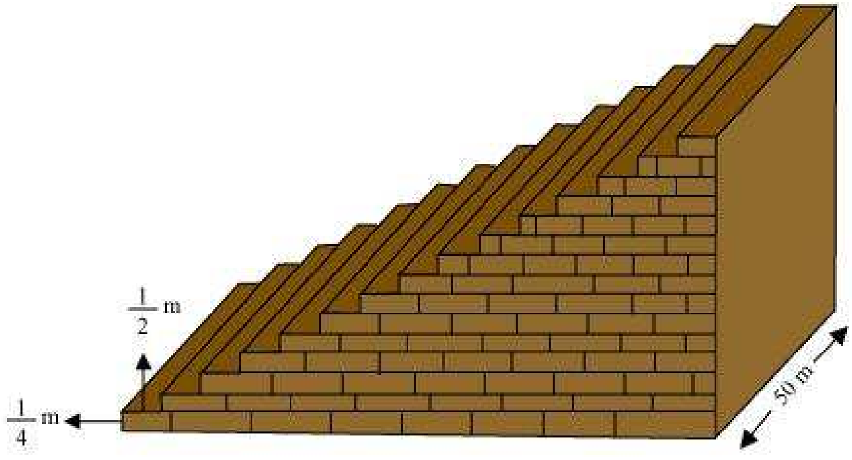
The value of x will be 35 only.

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

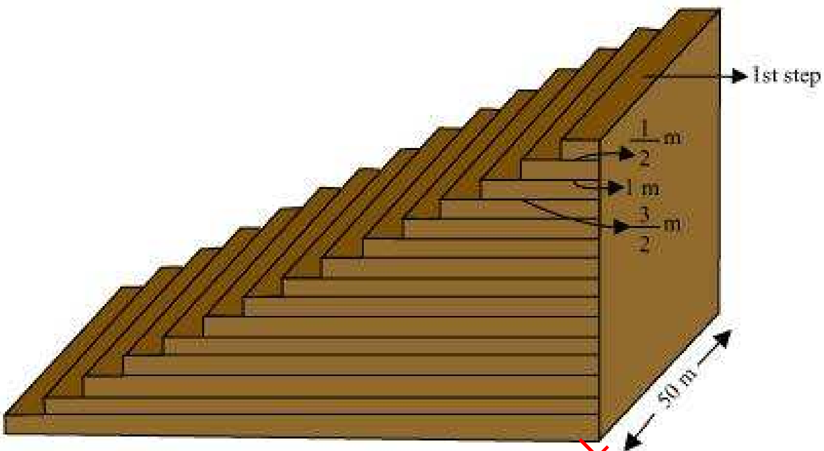
Question 5:

A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (See figure) calculate the total volume of concrete required to build the terrace.



Answer:



From the figure, it can be observed that

1^{st} step is $\frac{1}{2}$ m wide,

2^{nd} step is 1 m wide,

3^{rd} step is $\frac{3}{2}$ m wide.

Therefore, the width of each step is increasing by $\frac{1}{2}$ m each time whereas their height $\frac{1}{4}$ m and length 50 m remains the same.

Therefore, the widths of these steps are

$\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$

Volume of concrete in 1^{st} step $= \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$

Volume of concrete in 2^{nd} step $= \frac{1}{4} \times 1 \times 50 = \frac{25}{2}$

Volume of concrete in 3rd step $= \frac{1}{4} \times \frac{3}{2} \times 50 = \frac{75}{4}$

It can be observed that the volumes of concrete in these steps are in an A.P.

$$\frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \ldots$$

$$a = \frac{25}{4}$$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

$$\text{and } S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + (15-1)\frac{25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{(14)25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$$

$$= \frac{15}{2}(100) = 750$$

Volume of concrete required to build the terrace is 750 m^3.

Exercise 7.1

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1) (ii) (−5, 7), (−1, 3) (iii) (*a*, *b*), (− *a*, − *b*)

Answer:

(i) Distance between the two points is given by

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Therefore, distance between $(2, 3)$ and $(4, 1)$ is given by

$$l=\sqrt{(2-4)^2+(3-1)^2}=\sqrt{(-2)^2+(2)^2}$$
$$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

(ii) Distance between $(-5,7)$ and $(-1,3)$ is given by

$$l=\sqrt{(-5-(-1))^2+(7-3)^2}=\sqrt{(-4)^2+(4)^2}$$
$$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$$

(iii) Distance between (a,b) and $(-a,-b)$ is given by

$$l=\sqrt{(a-(-a))^2+(b-(-b))^2}$$
$$=\sqrt{(2a)^2+(2b)^2}=\sqrt{4a^2+4b^2}=2\sqrt{a^2+b^2}$$

Question 2:

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer:

Distance between points $(0,0)$ and $(36,15)$

$= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$

$= \sqrt{1296 + 225} = \sqrt{1521} = 39$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point (0, 0).

Therefore, town B will be at point (36, 15) with respect to town A.

And hence, as calculated above, the distance between town A and B will be 39 km.

Question 3:

Determine if the points (1, 5), (2, 3) and (− 2, − 11) are collinear.

Answer:

Let the points (1, 5), (2, 3), and (−2, −11) be representing the vertices A, B, and C of the given triangle respectively.

Let $A = (1,5), B = (2,3), C = (-2,-11)$

$\therefore AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$

$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16+196} = \sqrt{212}$

$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9+256} = \sqrt{265}$

Since $AB + BC \neq CA$,

Therefore, the points (1, 5), (2, 3), and (−2, −11) are not collinear.

Question 4:

Check whether (5, − 2), (6, 4) and (7, − 2) are the vertices of an isosceles triangle.

Answer:

Let the points (5, −2), (6, 4), and (7, −2) are representing the vertices A, B, and C of the given triangle respectively.

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

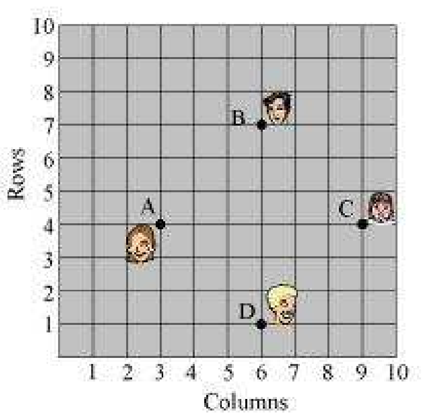
Therefore, AB = BC

As two sides are equal in length, therefore, ABCis an isosceles triangle.

Question 5:

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees.

Using distance formula, find which of them is correct.



Answer:

It can be observed that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of these 4 friends.

$$AB=\sqrt{(3-6)^2+(4-7)^2}=\sqrt{(-3)^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$$

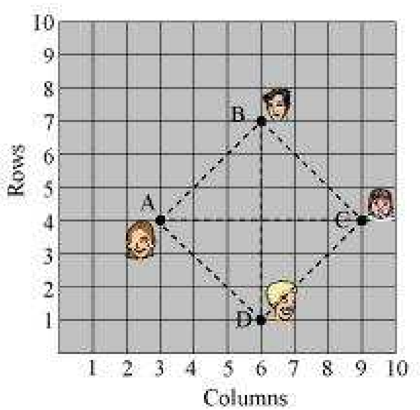
$$BC=\sqrt{(6-9)^2+(7-4)^2}=\sqrt{(-3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$$

$$CB=\sqrt{(9-6)^2+(4-1)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$$

$$AD=\sqrt{(3-6)^2+(4-1)^2}=\sqrt{(-3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$$

$$\text{Diagonal } AC=\sqrt{(3-9)^2+(4-4)^2}=\sqrt{(-6)^2+0^2}=6$$

$$\text{Diagonal } BD=\sqrt{(6-6)^2+(7-1)^2}=\sqrt{0^2+(6)^2}=6$$



It can be observed that all sides of this quadrilateral ABCD are of the same length and also the diagonals are of the same length.

Therefore, ABCD is a square and hence, Champa was correct

Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (− 1, − 2), (1, 0), (− 1, 2), (− 3, 0)

(ii) (− 3, 5), (3, 1), (0, 3), (− 1, − 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Answer:

(i) Let the points (−1, −2), (1, 0), (−1, 2), and (−3, 0) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\therefore AB=\sqrt{(-1-1)^2+(-2-0)^2}=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

$$BC=\sqrt{(1-(-1))^2+(0-2)^2}=\sqrt{(2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

$$CD=\sqrt{(-1-(-3))^2+(2-0)^2}=\sqrt{(2)^2+(2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

$$AD=\sqrt{(-1-(-3))^2+(-2-0)^2}=\sqrt{(2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

$$\text{Diagonal } AC=\sqrt{(-1-(-1))^2+(-2-2)^2}=\sqrt{0^2+(-4)^2}=\sqrt{16}=4$$

$$\text{Diagonal } BD=\sqrt{(1-(-3))^2+(0-0)^2}=\sqrt{(4)^2+0^2}=\sqrt{16}=4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii)Let the points (− 3, 5), (3, 1), (0, 3), and (−1, −4) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB=\sqrt{(-3-3)^2+(5-1)^2}=\sqrt{(-6)^2+(4)^2}=\sqrt{36+16}=\sqrt{52}=2\sqrt{13}$$

$$BC=\sqrt{(3-0)^2+(1-3)^2}=\sqrt{(3)^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$$

$$CD=\sqrt{(0-(-1))^2+(3-(-4))^2}=\sqrt{(1)^2+(7)^2}=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$$

$$AD=\sqrt{(-3-(-1))^2+(5-(-4))^2}=\sqrt{(-2)^2+(9)^2}=\sqrt{4+81}=\sqrt{85}$$

It can be observed that all sides of this quadrilateral are of different lengths. Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii)Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB=\sqrt{(4-7)^2+(5-6)^2}=\sqrt{(-3)^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}$$

$$BC=\sqrt{(7-4)^2+(6-3)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}$$

$$CD=\sqrt{(4-1)^2+(3-2)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$$

$$AD=\sqrt{(4-1)^2+(5-2)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}$$

$$\text{Diagonal AC}=\sqrt{(4-4)^2+(5-3)^2}=\sqrt{(0)^2+(2)^2}=\sqrt{0+4}=2$$

$$\text{Diagonal CD}=\sqrt{(7-1)^2+(6-2)^2}=\sqrt{(6)^2+(4)^2}=\sqrt{36+16}=\sqrt{52}=13\sqrt{2}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

Question 7:

Find the point on the *x*-axis which is equidistant from (2, − 5) and (− 2, 9).

Answer:

We have to find a point on *x*-axis. Therefore, its *y*-coordinate will be 0.

Let the point on *x*-axis be $(x,0)$.

$$\text{Distance between } (x,0) \text{ and } (2,-5)=\sqrt{(x-2)^2+(0-(-5))^2}=\sqrt{(x-2)^2+(5)^2}$$

$$\text{Distance between } (x,0) \text{ and } (-2,9)=\sqrt{(x-(-2))^2+(0-(-9))^2}=\sqrt{(x+2)^2+(9)^2}$$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2+(5)^2}=\sqrt{(x+2)^2+(9)^2}$$

$$(x-2)^2+25=(x+2)^2+81$$

$$x^2+4-4x+25=x^2+4+4x+81$$

$$8x=25-81$$

$$8x=-56$$

$$x=-7$$

Therefore, the point is (− 7, 0).

Question 8:

Find the values of *y* for which the distance between the points P (2, − 3) and Q (10, *y*) is 10 units.

Answer:

It is given that the distance between (2, −3) and (10, *y*) is 10.

$$\text{Therefore, } \sqrt{(2-10)^2+(-3-y)^2}=10$$

$$\sqrt{(-8)^2+(3+y)^2}=10$$

$$64+(y+3)^2=100$$

$$(y+3)^2=36$$

$$y+3=\pm 6$$

$$y+3=6 \text{ or } y+3=-6$$

$$\text{Therefore, } y=3 \text{ or } -9$$

Question 9:

If Q (0, 1) is equidistant from P (5, − 3) and R (*x*, 6), find the values of *x*. Also find the distance QR and PR.

Answer:

$$PQ=QR$$

$$\sqrt{(5-0)^2+(-3-1)^2}=\sqrt{(0-x)^2+(1-6)^2}$$

$$\sqrt{(5)^2+(-4)^2}=\sqrt{(-x)^2+(-5)^2}$$

$$\sqrt{25+16}=\sqrt{x^2+25}$$

$$41=x^2+25$$

$$16=x^2$$

$$x=\pm 4$$

Therefore, point R is (4, 6) or (−4, 6).

When point R is (4, 6),

$$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$$

$$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (−4, 6),

$$PR = \sqrt{(5-(-4))^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$

$$QR = \sqrt{(0-(-4))^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

Question 10:

Find a relation between *x* and *y* such that the point (*x*, *y*) is equidistant from the point (3, 6) and (− 3, 4).

Answer:

Point (*x*, *y*) is equidistant from (3, 6) and (−3, 4).

$$\therefore \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$36 - 16 = 6x + 6x + 12y - 8y$$

$$20 = 12x + 4y$$

$$3x + y = 5$$

$$3x + y - 5 = 0$$

Exercise 7.2

Question 1:

Find the coordinates of the point which divides the join of (− 1, 7) and (4, − 3) in the ratio 2:3.

Answer:

Let P(x, y) be the required point. Using the section formula, we obtain

$$x=\frac{2\times 4+3\times(-1)}{2+3}=\frac{8-3}{5}=\frac{5}{5}=1$$

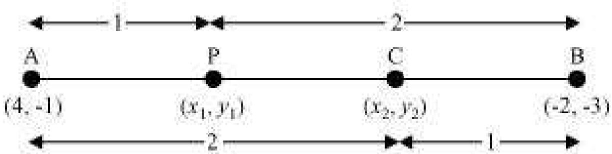
$$y=\frac{2\times(-3)+3\times 7}{2+3}=\frac{-6+21}{5}=\frac{15}{5}=3$$

Therefore, the point is (1, 3).

Question 2:

Find the coordinates of the points of trisection of the line segment joining (4, − 1) and (− 2, − 3).

Answer:



Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1=\frac{1\times(-2)+2\times 4}{1+2},\quad y_1=\frac{1\times(-3)+2\times(-1)}{1+2}$$

$$x_1=\frac{-2+8}{3}=\frac{6}{3}=2,\quad y_1=\frac{-3-2}{3}=\frac{-5}{3}$$

$$\text{Therefore, } P(x_1, y_1)=\left(2,-\frac{5}{3}\right)$$

Point Q divides AB internally in the ratio 2:1.

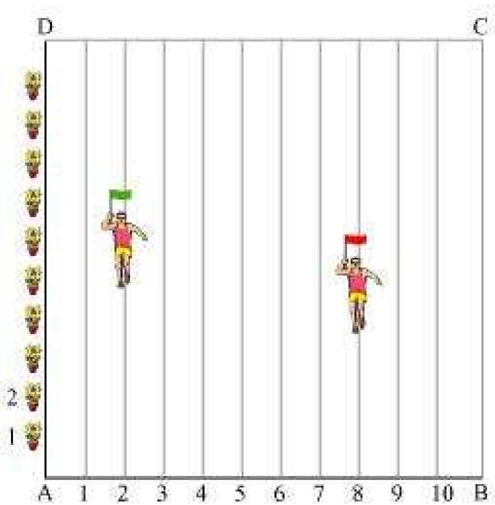
$$x_2=\frac{2\times(-2)+1\times4}{2+1},\quad y_2=\frac{2\times(-3)+1\times(-1)}{2+1}$$

$$x_2=\frac{-4+4}{3}=0,\qquad y_2=\frac{-6-1}{3}=\frac{-7}{3}$$

$$Q(x_2,y_2)=\left(0,-\frac{7}{3}\right)$$

Question 3:

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer:

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\left(\frac{1}{4}\times 100\right)\text{m} = 25$ m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., $\left(\frac{1}{5}\times 100\right)\text{m} = 20$ m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$$= \sqrt{(8-2)^2+(25-20)^2} = \sqrt{36+25} = \sqrt{61}\text{ m}$$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (*x*, *y*).

$$x = \frac{2+8}{2},\quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5,\ y = \frac{45}{2} = 22.5$$

$$\text{Hence, } A(x, y) = (5, 22.5)$$

Therefore, Rashmi should post her blue flag at 22.5m on 5th line

Question 4:

Find the ratio in which the line segment joining the points (− 3, 10) and (6, − 8) is divided by (− 1, 6).

Answer:

Let the ratio in which the line segment joining (−3, 10) and (6, −8) is divided by point (−1, 6) be *k* : 1.

Therefore, $-1=\frac{6k-3}{k+1}$

$-k-1=6k-3$

$7k=2$

$k=\frac{2}{7}$

Therefore, the required ratio is $2:7$.

Question 5:

Find the ratio in which the line segment joining A (1, − 5) and B (− 4, 5) is divided by the *x*-axis. Also find the coordinates of the point of division.

Answer:

Let the ratio in which the line segment joining A (1, −5) and B (−4, 5) is divided by *x*-axisbe $k:1$.

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1},\frac{5k-5}{k+1}\right)$.

We know that *y*-coordinate of any point on *x*-axis is 0.

$\therefore \frac{5k-5}{k+1}=0$

$k=1$

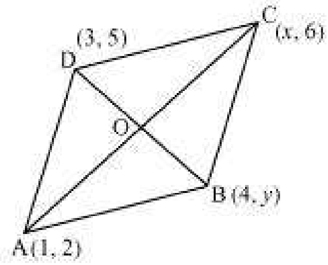
Therefore, *x*-axis divides it in the ratio 1:1.

Division point = $\left(\frac{-4(1)+1}{1+1},\frac{5(1)-5}{1+1}\right)=\left(\frac{-4+1}{2},\frac{5-5}{2}\right)=\left(\frac{-3}{2},0\right)$

Question 6:

If (1, 2), (4, *y*), (*x*, 6) and (3, 5) are the vertices of a parallelogram taken in order, find *x* and *y*.

Answer:



Let (1, 2), (4, y), (x, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2},\frac{2+6}{2}\right)\Rightarrow\left(\frac{x+1}{2},4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2},\frac{5+y}{2}\right)\Rightarrow\left(\frac{7}{2},\frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore \frac{x+1}{2}=\frac{7}{2} \text{ and } 4=\frac{5+y}{2}$$

$$\Rightarrow x+1=7 \text{ and } 5+y=8$$

$$\Rightarrow x=6 \text{ and } y=3$$

Question 7:

Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, − 3) and B is (1, 4)

Answer:

Let the coordinates of point A be (x, y).

Mid-point of AB is (2, −3), which is the center of the circle.

$$\therefore (2,-3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2} = 2 \text{ and } \frac{y+4}{2} = -3$$

$$\Rightarrow x+1 = 4 \text{ and } y+4 = -6$$

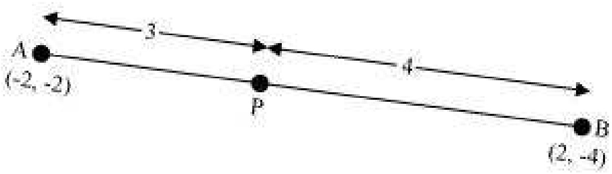
$$\Rightarrow x = 3 \text{ and } y = -10$$

Therefore, the coordinates of A are $(3,-10)$.

Question 8:

If A and B are (− 2, − 2) and (2, − 4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

Answer:



The coordinates of point A and B are (−2, −2) and (2, −4) respectively.

Since $AP = \frac{3}{7}AB$,

Therefore, AP: PB = 3:4

Point P divides the line segment AB in the ratio 3:4.

$$\text{Coordinates of P} = \left(\frac{3\times 2+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right)$$

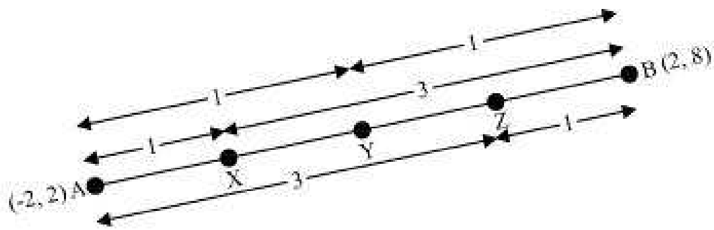
$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

$$= \left(-\frac{2}{7}, -\frac{20}{7}\right)$$

Question 9:

Find the coordinates of the points which divide the line segment joining A (− 2, 2) and B (2, 8) into four equal parts.

Answer:



From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\text{Coordinates of P} = \left(\frac{1\times 2+3\times(-2)}{1+3}, \frac{1\times 8+3\times 2}{1+3}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

$$\text{Coordinates of Q} = \left(\frac{2+(-2)}{2}, \frac{2+8}{2}\right)$$

$$= (0, 5)$$

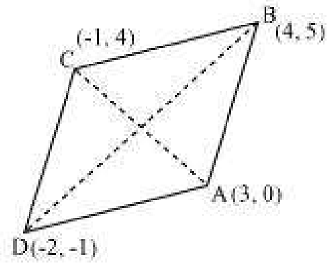
$$\text{Coordinates of R} = \left(\frac{3\times 2+1\times(-2)}{3+1}, \frac{3\times 8+1\times 2}{3+1}\right)$$

$$= \left(1, \frac{13}{2}\right)$$

Question 10:

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (− 1, 4) and (− 2, −1) taken in order. [**Hint:** Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer:



Let (3, 0), (4, 5), (−1, 4) and (−2, −1) are the vertices A, B, C, D of a rhombus ABCD.

$$\text{Length of diagonal AC} = \sqrt{\left[3-(-1)\right]^2 + (0-4)^2}$$
$$= \sqrt{16+16} = 4\sqrt{2}$$

$$\text{Length of diagonal BD} = \sqrt{\left[4-(-2)\right]^2 + \left[5-(-1)\right]^2}$$
$$= \sqrt{36+36} = 6\sqrt{2}$$

$$\text{Therefore, area of rhombus ABCD} = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$
$$= 24 \text{ square units}$$

Exercise 7.3

Question 1:

Find the area of the triangle whose vertices are:

(i) (2, 3), (− 1, 0), (2, − 4) (ii) (− 5, − 1), (3, − 5), (5, 2)

Answer:

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right\}$$

$$\text{Area of the given triangle} = \frac{1}{2}\left[2\left\{0-(-4)\right\} + (-1)\left\{(-4)-(3)\right\} + 2(3-0)\right]$$

$$= \frac{1}{2}\left\{8+7+6\right\}$$

$$= \frac{21}{2} \text{ square units}$$

(ii) $$\text{Area of the given triangle} = \frac{1}{2}\left[(-5)\left\{(-5)-(2)\right\} + 3\left(2-(-1)\right) + 5\left\{-1-(-5)\right\}\right]$$

$$= \frac{1}{2}\left\{35+9+20\right\}$$

$$= 32 \text{ square units}$$

Question 2:

In each of the following find the value of '*k*', for which the points are collinear.

(i) (7, − 2), (5, 1), (3, − *k*) (ii) (8, 1), (*k*, − 4), (2, − 5)

Answer:

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, −2) (5, 1), and (3, *k*), area = 0

$$\frac{1}{2}\left[7\{1-k\}+5\{k-(-2)\}+3\{(-2)-1\}\right]=0$$

$$7-7k+5k+10-9=0$$

$$-2k+8=0$$

$$k=4$$

(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points (8, 1), (k, −4), and (2, −5), area = 0

$$\frac{1}{2}\left[8\{-4-(-5)\}+k\{(-5)-(1)\}+2\{1-(-4)\}\right]=0$$

$$8-6k+10=0$$

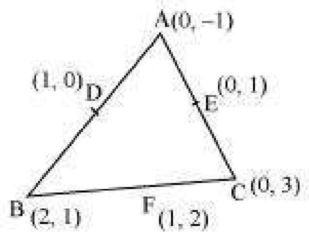
$$6k=18$$

$$k=3$$

Question 3:

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, − 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer:



Let the vertices of the triangle be A (0, −1), B (2, 1), C (0, 3).

Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by

$$D=\left(\frac{0+2}{2},\frac{-1+1}{2}\right)=(1,0)$$

$$E=\left(\frac{0+0}{2},\frac{3-1}{2}\right)=(0,1)$$

$$F=\left(\frac{2+0}{2},\frac{1+3}{2}\right)=(1,2)$$

$$\text{Area of a triangle}=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

$$\text{Area of }\Delta DEF=\frac{1}{2}\{1(2-1)+1(1-0)+0(0-2)\}$$

$$=\frac{1}{2}(1+1)=1\text{ square units}$$

$$\text{Area of }\Delta ABC=\frac{1}{2}\left[0(1-3)+2\{3-(-1)\}+0(-1-1)\right]$$

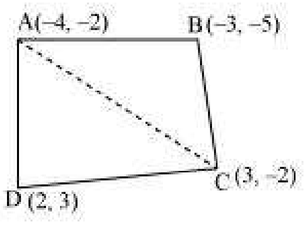
$$=\frac{1}{2}\{8\}=4\text{ square units}$$

Therefore, required ratio $=1:4$

Question 4:

Find the area of the quadrilateral whose vertices, taken in order, are (− 4, − 2), (− 3, − 5), (3, − 2) and (2, 3)

Answer:



Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD.

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[(-4)\{(-5)-(-2)\} + (-3)\{(-2)-(-2)\} + 3\{(-2)-(-5)\}\right]$$

$$= \frac{1}{2}(12 + 0 + 9) = \frac{21}{2} \text{ square units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left[(-4)\{(-2)-(3)\} + 3\{(3)-(-2)\} + 2\{(-2)-(-2)\}\right]$$

$$= \frac{1}{2}\{20 + 15 + 0\} = \frac{35}{2} \text{ square units}$$

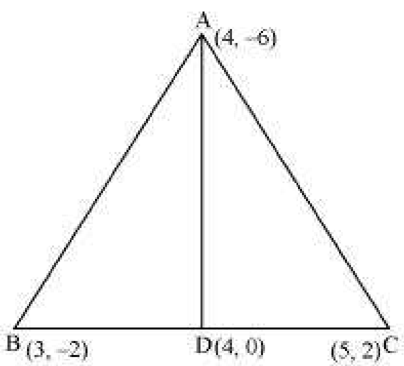
$$\text{Area of } \square ABCD = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \left(\frac{21}{2} + \frac{35}{2}\right) \text{square units} = 28 \text{ square units}$$

Question 5:

You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, − 6), B (3, − 2) and C (5, 2)

Answer:



Let the vertices of the triangle be A (4, −6), B (3, −2), and C (5, 2).

Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.

$$\text{Coordinates of point D} = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$$

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta\text{ABD} = \frac{1}{2}\left[(4)\{(-2)-(0)\} + (3)\{(0)-(-6)\} + (4)\{(-6)-(-2)\}\right]$$

$$= \frac{1}{2}(-8 + 18 - 16) = -3 \text{ square units}$$

However, area cannot be negative. Therefore, area of ΔABD is 3 square units.

$$\text{Area of } \Delta\text{ADC} = \frac{1}{2}\left[(4)\{0-(2)\} + (4)\{(2)-(-6)\} + (5)\{(-6)-(0)\}\right]$$

$$= \frac{1}{2}(-8 + 32 - 30) = -3 \text{ square units}$$

However, area cannot be negative. Therefore, area of ΔADC is 3 square units.

Clearly, median AD has divided ΔABC in two triangles of equal areas.

Exercise 7.4

Question 1:

Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, − 2) and B(3, 7)

Answer:

Let the given line divide the line segment joining the points A(2, −2) and B(3, 7) in a ratio k : 1.

Coordinates of the point of division $= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

This point also lies on $2x + y - 4 = 0$

$$\therefore 2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow \frac{6k+4+7k-2-4k-4}{k+1}=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, −2) and B(3, 7) is 2:9.

Question 2:

Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.

Answer:

If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right\}$$

$$\text{Area} = \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]$$
$$0 = \frac{1}{2}\left[2x - y + 7y - 14\right]$$
$$0 = \frac{1}{2}\left[2x + 6y - 14\right]$$
$$2x + 6y - 14 = 0$$
$$x + 3y - 7 = 0$$

This is the required relation between *x* and *y*.

Question 3:

Find the centre of a circle passing through the points (6, − 6), (3, − 7) and (3, 3).

Answer:

Let O (*x*, *y*) be the centre of the circle. And let the points (6, −6), (3, −7), and (3, 3) be representing the points A, B, and C on the circumference of the circle.

$$\therefore \text{OA} = \sqrt{(x-6)^2 + (y+6)^2}$$
$$\text{OB} = \sqrt{(x-3)^2 + (y+7)^2}$$
$$\text{OC} = \sqrt{(x-3)^2 + (y-3)^2}$$

However, $\text{OA} = \text{OB}$ (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$
$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$$
$$\Rightarrow -6x - 2y + 14 = 0$$
$$\Rightarrow 3x + y = 7 \qquad \ldots (1)$$

Similarly, $\text{OA} = \text{OC}$ (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$
$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$$
$$\Rightarrow -6x + 18y + 54 = 0$$
$$\Rightarrow -3x + 9y = -27 \qquad \ldots (2)$$

On adding equation (1) and (2), we obtain

$10y = -20$

$y = -2$

From equation (1), we obtain

$3x - 2 = 7$

$3x = 9$

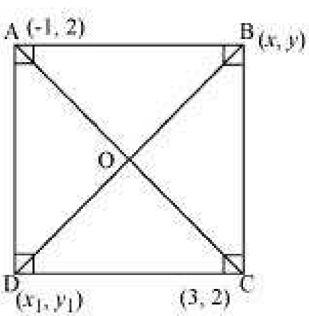
$x = 3$

Therefore, the centre of the circle is (3, −2).

Question 4:

The two opposite vertices of a square are (−1, 2) and (3, 2). Find the coordinates of the other two vertices.

Answer:



Let ABCD be a square having (−1, 2) and (3, 2) as vertices A and C respectively. Let (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively.

We know that the sides of a square are equal to each other.

$\therefore$ AB = BC

$$\Rightarrow \sqrt{(x+1)^2+(y-2)^2} = \sqrt{(x-3)^2+(y-2)^2}$$

$$\Rightarrow x^2+2x+1+y^2-4y+4 = x^2+9-6x+y^2+4-4y$$

$$\Rightarrow 8x = 8$$

$$\Rightarrow x = 1$$

We know that in a square, all interior angles are of 90°.

In ΔABC,

$AB^2 + BC^2 = AC^2$

$$\Rightarrow \left(\sqrt{(1+1)^2+(y-2)^2}\right)^2+\left(\sqrt{(1-3)^2+(y-2)^2}\right)^2=\left(\sqrt{(3+1)^2+(2-2)^2}\right)^2$$

$\Rightarrow 4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

$\Rightarrow 2y^2 + 16 - 8y = 16$

$\Rightarrow 2y^2 - 8y = 0$

$\Rightarrow y(y - 4) = 0$

$\Rightarrow y = 0$ or 4

We know that in a square, the diagonals are of equal length and bisect each other at 90°. Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

$$\text{Coordinate of point O} = \left(\frac{-1+3}{2}, \frac{2+2}{2}\right)$$

$$\left(\frac{1+x_1}{2}, \frac{y+y_1}{2}\right) = (1, 2)$$

$$\frac{1+x_1}{2} = 1$$

$$1 + x_1 = 2$$

$$x_1 = 1$$

$$\text{and } \frac{y+y_1}{2} = 2$$

$\Rightarrow y + y_1 = 4$

If $y = 0$,

$y_1 = 4$

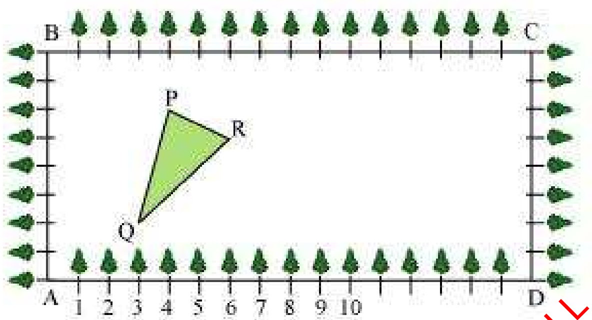
If $y = 4$,

$y_1 = 0$

Therefore, the required coordinates are (1, 0) and (1, 4).

Question 5:

The class X students of a secondary school in Krishinagar have been allotted a on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

Answer:

(i) Taking A as origin, we will take AD as *x*-axis and AB as *y*-axis. It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.

$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[4(2-5) + 3(5-6) + 6(6-2)\right]$$

$$= \frac{1}{2}[-12 - 3 + 24]$$

$$= \frac{9}{2} \text{ square units}$$

(ii) Taking C as origin, CB as *x*-axis, and CD as *y*-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.

$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 + y_2\right)\right]$$

$$= \frac{1}{2}\left[12(6-3) + 13(3-2) + 10(2-6)\right]$$

$$= \frac{1}{2}\left[36 + 13 - 40\right]$$

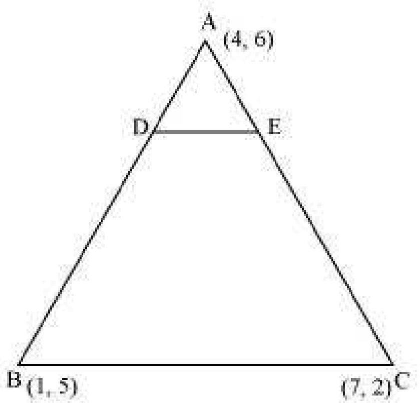
$$= \frac{9}{2}\text{square units}$$

It can be observed that the area of the triangle is same in both the cases.

Question 6:

The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC. (Recall Converse of basic proportionality theorem and Theorem 6.6 related to ratio of areas of two similar triangles)

Answer:



Given that, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$$\frac{AD}{AD+DB}=\frac{AE}{AE+EC}=\frac{1}{4}$$

$$\frac{AD}{DB}=\frac{AE}{EC}=\frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

$$\text{Coordinates of Point D}=\left(\frac{1\times1+3\times4}{1+3},\frac{1\times5+3\times6}{1+3}\right)$$

$$=\left(\frac{13}{4},\frac{23}{4}\right)$$

$$\text{Coordinates of point E }=\left(\frac{1\times7+3\times4}{1+3},\frac{1\times2+3\times6}{1+3}\right)$$

$$=\left(\frac{19}{4},\frac{20}{4}\right)$$

$$\text{Area of a triangle}=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$\text{Area of }\Delta ADE=\frac{1}{2}\left[4\left(\frac{23}{4}-\frac{20}{4}\right)+\frac{13}{4}\left(\frac{20}{4}-6\right)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$$

$$=\frac{1}{2}\left[3-\frac{13}{4}+\frac{19}{16}\right]=\frac{1}{2}\left[\frac{48-52+19}{16}\right]=\frac{15}{32}\text{square units}$$

$$\text{Area of }\Delta ABC=\frac{1}{2}\left[4(5-2)+1(2-6)+7(6-5)\right]$$

$$=\frac{1}{2}[12-4+7]=\frac{15}{2}\text{square units}$$

Clearly, the ratio between the areas of ΔADE and ΔABC is 1:16.

Alternatively,

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

Therefore, ratio between the areas of ΔADE and ΔABC = $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Question 7:

Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of point D.

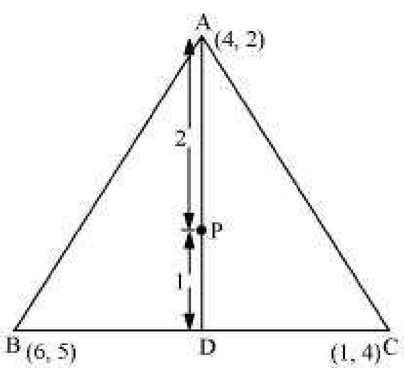
(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1

(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.

(iv) What do you observe?

(v) If A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Answer:



(i) Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii) Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of P} = \left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of E} = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of Q} = \left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB.

$$\text{Coordinates of F} = \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of R} = \left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC, having its vertices as A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3).

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

Let the centroid of this triangle be O.

Point O divides the side AD in a ratio 2:1.

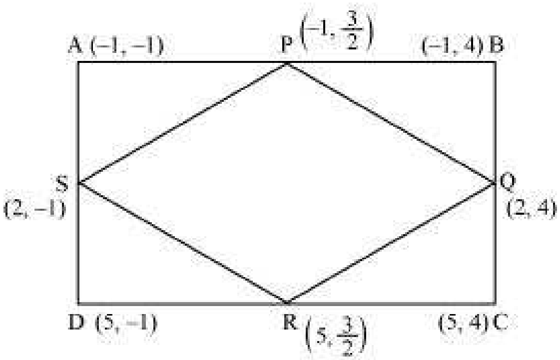
$$\text{Coordinates of O} = \left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Question 8:

ABCD is a rectangle formed by the points A (− 1, − 1), B (− 1, 4), C (5, 4) and D (5, − 1). P, Q, R and S are the mid-points of AB, BC, CD, and DA respectively. Is the quadrilateral PQRS is a square? a rectangle? or a rhombus? Justify your answer.

Answer:



P is the mid-point of side AB.

Therefore, the coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Similarly, the coordinates of Q, R, and S are $(2, 4)$, $\left(5, \frac{3}{2}\right)$, and $(2, -1)$ respectively.

$$\text{Length of PQ} = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of QR} = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of RS} = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of PR} = \sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$$

$$\text{Length of QS} = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.

Exercise 6.1

Question 1:

Fill in the blanks using correct word given in the brackets:−

(i) All circles are ___________. (congruent, similar)

(ii) All squares are ___________. (similar, congruent)

(iii) All ___________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ___________ and (b) their corresponding sides are ___________. (equal, proportional)

Answer:

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

Question 2:

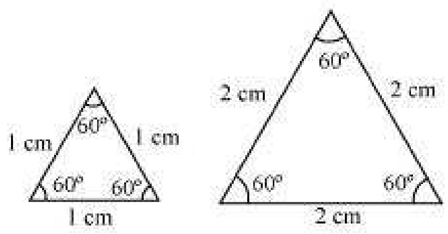
Give two different examples of pair of

(i) Similar figures

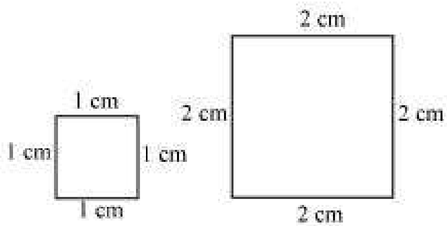
(ii)Non-similar figures

Answer:

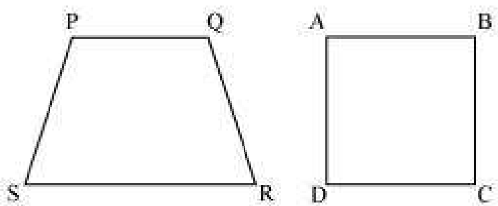
(i) Two equilateral triangles with sides 1 cm and 2 cm



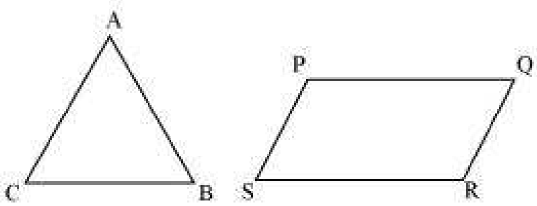
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

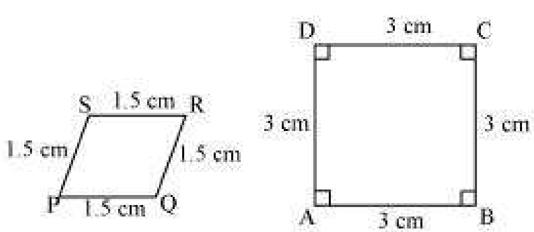


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer:

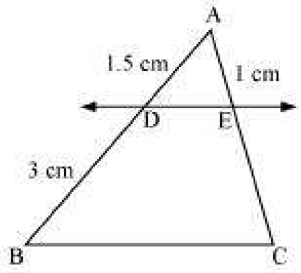
Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Exercise 6.2

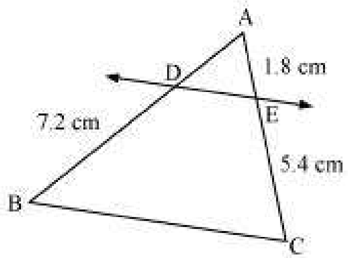
Question 1:

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i)

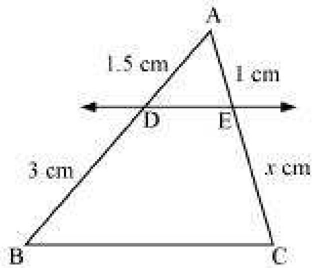


(ii)



Answer:

(i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB}=\frac{AE}{EC}$$

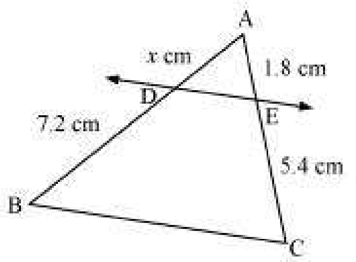
$$\frac{1.5}{3}=\frac{1}{x}$$

$$x=\frac{3\times 1}{1.5}$$

$$x=2$$

$$\therefore EC=2\text{ cm}$$

(ii)



Let AD = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB}=\frac{AE}{EC}$$

$$\frac{x}{7.2}=\frac{1.8}{5.4}$$

$$x=\frac{1.8\times 7.2}{5.4}$$

$$x=2.4$$

$$\therefore AD=2.4\text{ cm}$$

Question 2:

E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

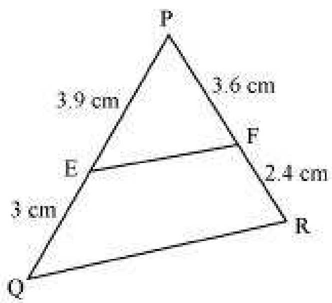
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer:

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

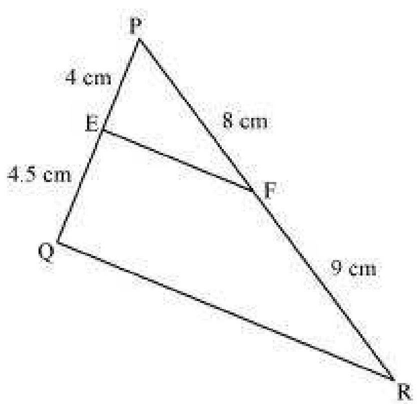
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

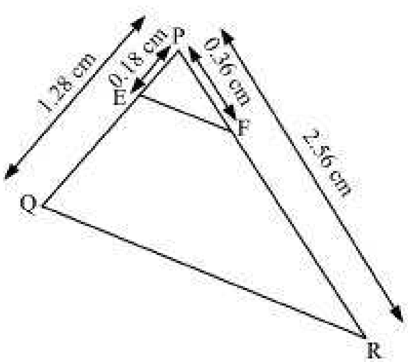
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

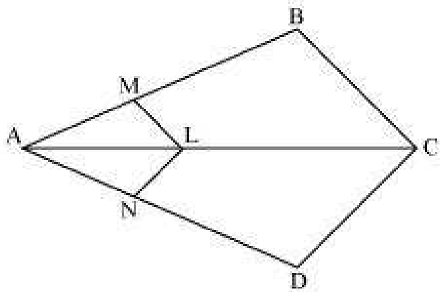
Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

Therefore, EF is parallel to QR.

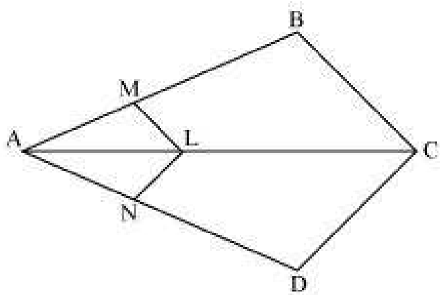
Question 3:

In the following figure, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}.$$



Answer:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \qquad (i)$$

Similarly, LN || CD

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \qquad (ii)$$

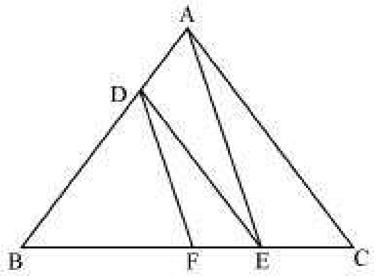
From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

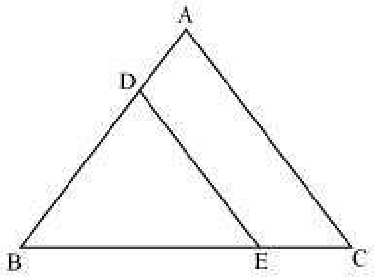
Question 4:

In the following figure, DE || AC and DF || AE. Prove that

$\frac{BF}{FE}=\frac{BE}{EC}$.

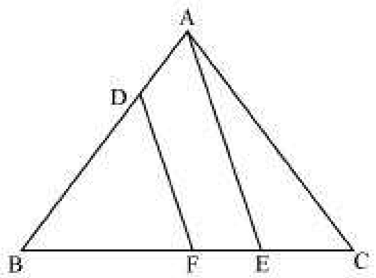


Answer:



In ΔABC, DE || AC

$$\therefore \frac{BD}{DA}=\frac{BE}{EC} \qquad \text{(Basic Proportionality Theorem)} \qquad (i)$$



In ΔBAE, $DF \parallel AE$

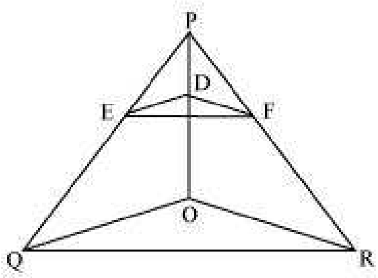
$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ (Basic Proportionality Theorem) (ii)

From (i) and (ii), we obtain

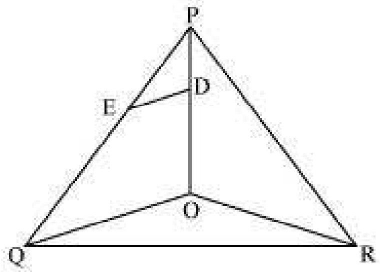
$\frac{BE}{EC} = \frac{BF}{FE}$

Question 5:

In the following figure, DE || OQ and DF || OR, show that EF || QR.

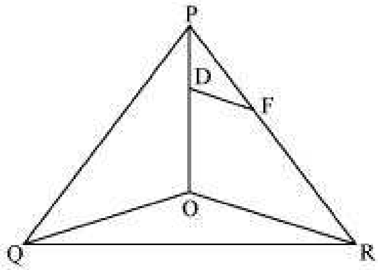


Answer:



In Δ POQ, DE || OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ (Basic proportionality theorem) (i)



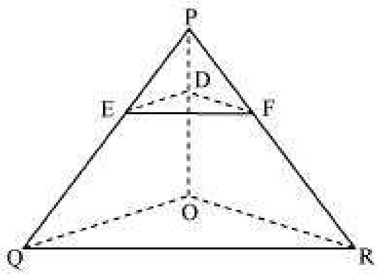
In ΔPOR, $DF \parallel OR$

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we obtain

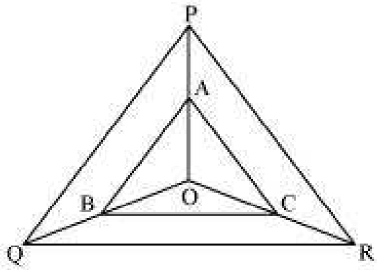
$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

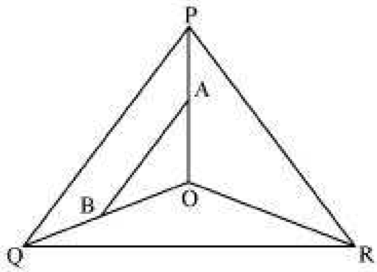


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

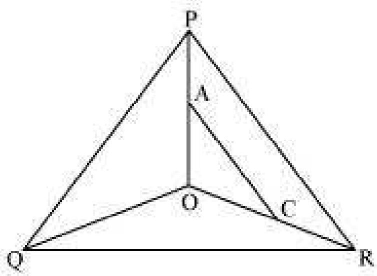


Answer:



In Δ POQ, AB || PQ

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ (Basic proportionality theorem) (i)



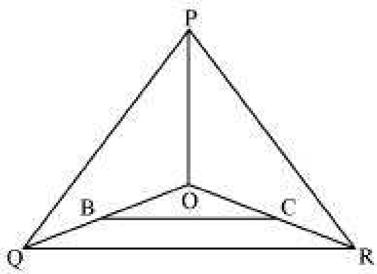
In ΔPOR, $AC \parallel PR$

$\therefore \dfrac{OA}{AP} = \dfrac{OC}{CR}$ (By basic proportionality theorem) (ii)

From (i) and (ii), we obtain

$\dfrac{OB}{BQ} = \dfrac{OC}{CR}$

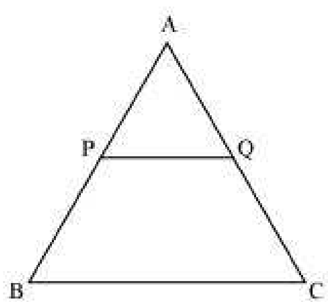
$\therefore BC \parallel QR$ (By the converse of basic proportionality theorem)



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad \left(\text{P is the mid-point of AB.} \ \therefore \ AP = PB\right)$$

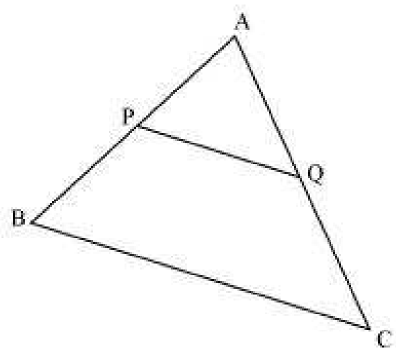
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC

It can be observed that

$\frac{AP}{PB} = \frac{1}{1}$

and $\frac{AQ}{QC} = \frac{1}{1}$

$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$

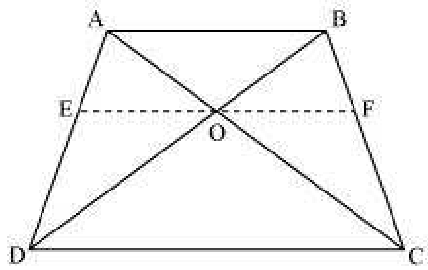
Hence, by using basic proportionality theorem, we obtain

$PQ \| BC$

Question 9:

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer:



Draw a line EF through point O, such that $EF \| CD$

In ΔADC, $EO \| CD$

By using basic proportionality theorem, we obtain

$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$

In ΔABD, $OE \| AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE}=\frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED}=\frac{BO}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC}=\frac{BO}{OD}$$

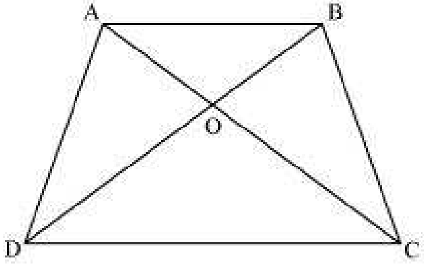
$$\Rightarrow \frac{AO}{BO}=\frac{OC}{OD}$$

Question 10:

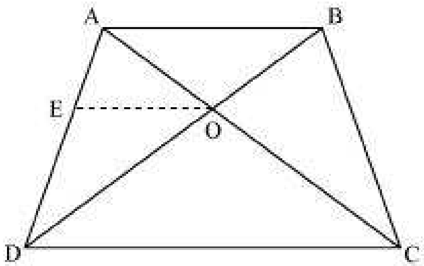
The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO}=\frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer:

Let us consider the following figure for the given question.



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

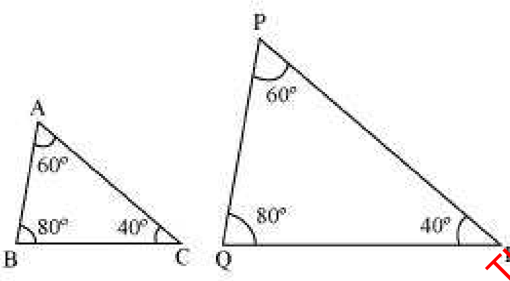
∴ ABCD is a trapezium.

Exercise 6.3

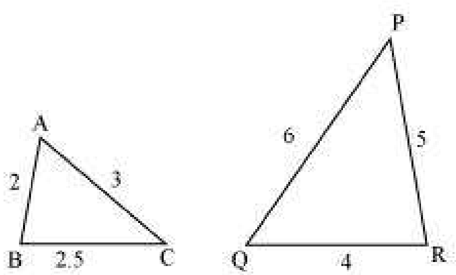
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

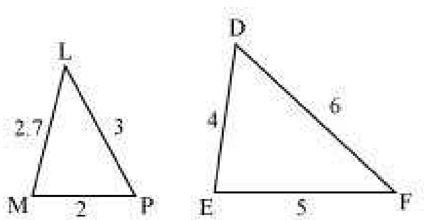
(i)



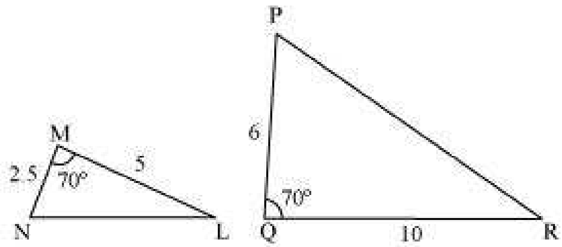
(ii)



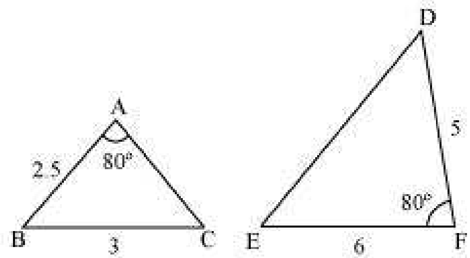
(iii)



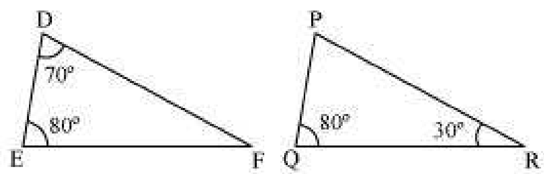
(iv)



(v)



(vi)



Answer:

(i) $\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Therefore, $\Delta ABC \sim \Delta PQR$ [By AAA similarity criterion]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii)

$\therefore\ \Delta ABC \sim \Delta QRP$ [By SSS similarity criterion]

(iii)The given triangles are not similar as the corresponding sides are not proportional.

(iv)The given triangles are not similar as the corresponding sides are not proportional.

(v)The given triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF,

$\angle D + \angle E + \angle F = 180^\circ$

(Sum of the measures of the angles of a triangle is 180º.)

$70^\circ + 80^\circ + \angle F = 180^\circ$

$\angle F = 30^\circ$

Similarly, in ΔPQR,

$\angle P + \angle Q + \angle R = 180^\circ$

(Sum of the measures of the angles of a triangle is 180º.)

$\angle P + 80^\circ + 30^\circ = 180^\circ$

$\angle P = 70^\circ$

In ΔDEF and ΔPQR,

$\angle D = \angle P$ (Each 70°)

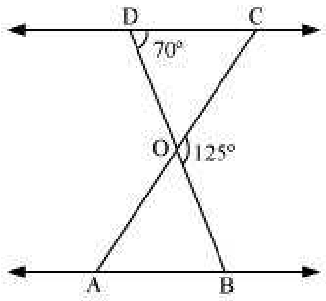
$\angle E = \angle Q$ (Each 80°)

$\angle F = \angle R$ (Each 30°)

∴ ΔDEF ~ ΔPQR [By AAA similarity criterion]

Question 2:

In the following figure, ΔODC ~ ΔOBA, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer:

DOB is a straight line.

$\therefore \angle DOC + \angle COB = 180°$

$\Rightarrow \angle DOC = 180° - 125°$

$= 55°$

In ΔDOC,

$\angle DCO + \angle CDO + \angle DOC = 180°$

(Sum of the measures of the angles of a triangle is 180º.)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO = 55°$

It is given that $\Delta ODC \sim \Delta OBA$.

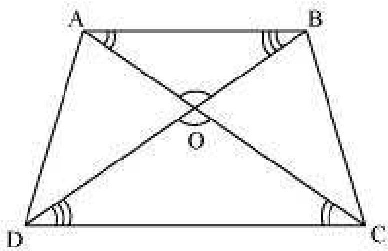
$\therefore \angle OAB = \angle OCD$ [Corresponding angles are equal in similar triangles.]

$\Rightarrow \angle OAB = 55°$

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer:



In ΔDOC and ΔBOA,

$\angle$CDO = $\angle$ABO [Alternate interior angles as AB || CD]

$\angle$DCO = $\angle$BAO [Alternate interior angles as AB || CD]

$\angle$DOC = $\angle$BOA [Vertically opposite angles]

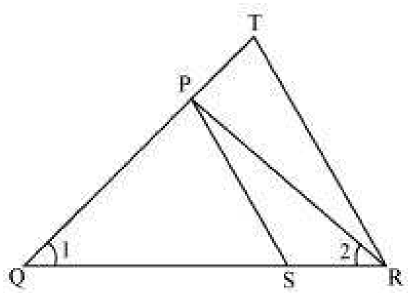
∴ ΔDOC ~ ΔBOA [AAA similarity criterion]

$\therefore \frac{DO}{BO} = \frac{OC}{OA}$ [Corresponding sides are proportional]

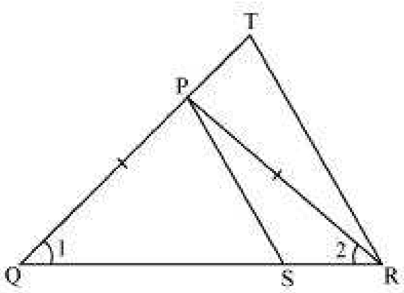
$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2.$ Show that $\Delta PQS \sim \Delta TQR$



Answer:



In ΔPQR, ∠PQR = ∠PRQ

∴ PQ = PR (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad (ii)$$

In ΔPQS and ΔTQR,

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad [\text{Using } (ii)]$$

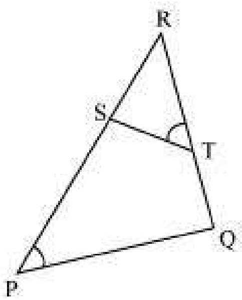
$$\angle Q = \angle Q$$

$$\therefore \ \Delta PQS \sim \Delta TQR \qquad [\text{SAS similarity criterion}]$$

Question 5:

S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Answer:



In ΔRPQ and ΔRST,

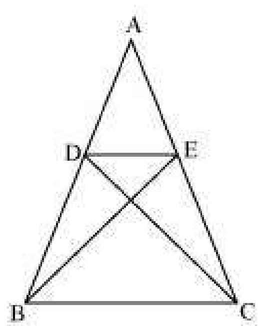
$\angle$RTS = $\angle$QPS (Given)

$\angle$R = $\angle$R (Common angle)

∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

Question 6:

In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Answer:

It is given that ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] (1)

And, AD = AE [By CPCT] (2)

In ΔADE and ΔABC,

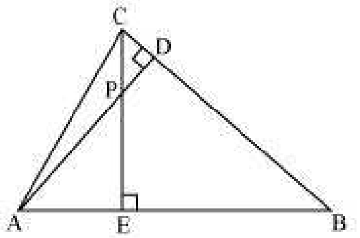
$\frac{AD}{AB} = \frac{AE}{AC}$ [Dividing equation (2) by (1)]

$\angle$A = $\angle$A [Common angle]

∴ ΔADE ~ ΔABC [By SAS similarity criterion]

Question 7:

In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) ΔAEP ~ ΔCDP

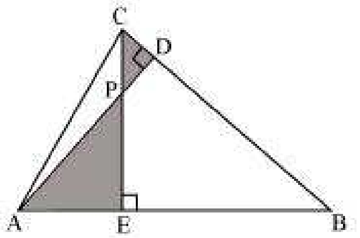
(ii) ΔABD ~ ΔCBE

(iii) ΔAEP ~ ΔADB

(v) ΔPDC ~ ΔBEC

Answer:

(i)



In ΔAEP and ΔCDP,

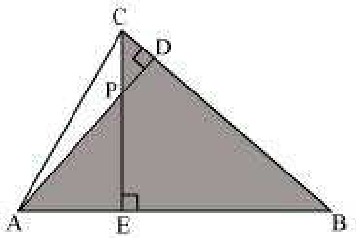
∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔCDP

(ii)



In ΔABD and ΔCBE,

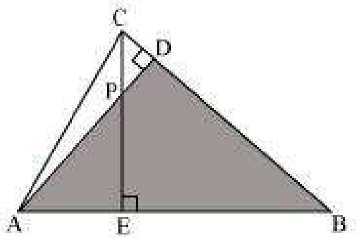
∠ADB = ∠CEB (Each 90°)

∠ABD = ∠CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ~ ΔCBE

(iii)



In ΔAEP and ΔADB,

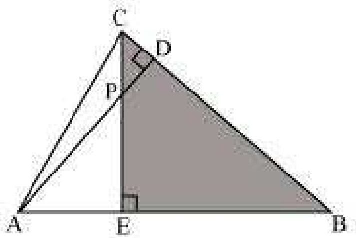
∠AEP = ∠ADB (Each 90°)

∠PAE = ∠DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔADB

(iv)



In ΔPDC and ΔBEC

$\angle PDC = \angle BEC$ (Each 90°)

$\angle PCD = \angle BCE$ (Common angle)

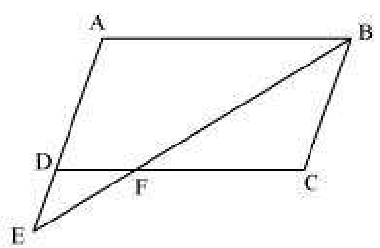
Hence, by using AA similarity criterion,

$\Delta PDC \sim \Delta BEC$

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$

Answer:



In ΔABE and ΔCFB,

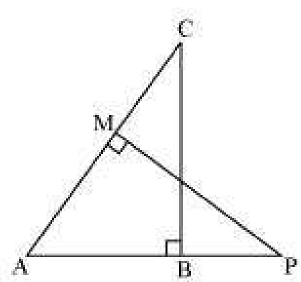
$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles as AE || BC)

$\therefore \Delta ABE \sim \Delta CFB$ (By AA similarity criterion)

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer:

In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ ΔABC ~ ΔAMP (By AA similarity criterion)

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides of similar triangles are proportional)

Question 10:

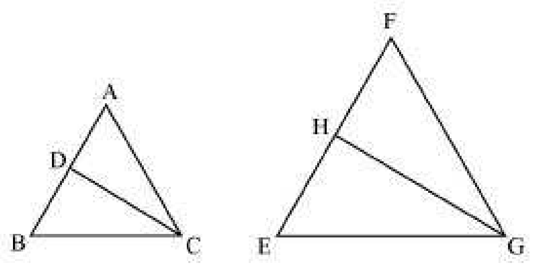
CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) ΔDCB ~ ΔHGE

(iii) ΔDCA ~ ΔHGF

Answer:



It is given that ΔABC ~ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ ΔACD ~ ΔFGH (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In ΔDCB and ΔHGE,

∠DCB = ∠HGE (Proved above)

∠B = ∠E (Proved above)

∴ ΔDCB ~ ΔHGE (By AA similarity criterion)

In ΔDCA and ΔHGF,

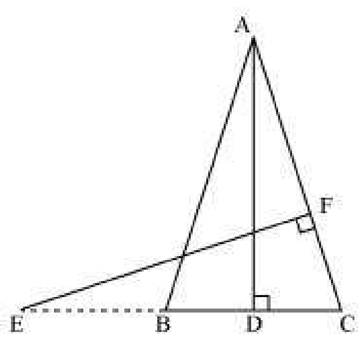
∠ACD = ∠FGH (Proved above)

∠A = ∠F (Proved above)

∴ ΔDCA ~ ΔHGF (By AA similarity criterion)

Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF



Answer:

It is given that ABC is an isosceles triangle.

∴ AB = AC

⇒ ∠ABD = ∠ECF

In ΔABD and ΔECF,

∠ADB = ∠EFC (Each 90°)

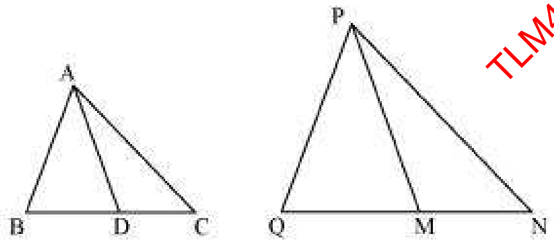
∠BAD = ∠CEF (Proved above)

∴ ΔABD ~ ΔECF (By using AA similarity criterion)

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that ΔABC ~ ΔPQR.

Answer:



Median divides the opposite side.

∴ $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (Proved above)

∴ ΔABD ~ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

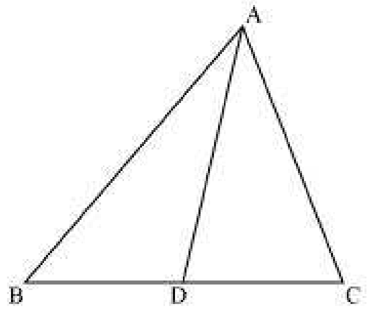
$\frac{AB}{PQ} = \frac{BC}{QR}$

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that $CA^2 = CB.CD.$

Answer:



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Given)

∠ACD = ∠BCA (Common angle)

∴ ΔADC ~ ΔBAC (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

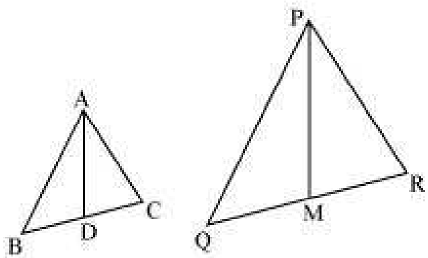
$\therefore \frac{CA}{CB} = \frac{CD}{CA}$

$\Rightarrow CA^2 = CB \times CD$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

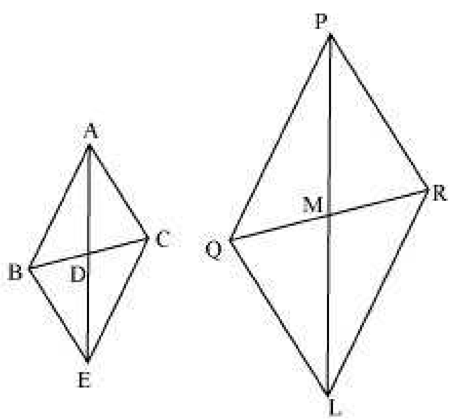
Answer:



Given that,

$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BE}{QL}=\frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BE}{QL}=\frac{AE}{PL}$$

∴ ΔABE ~ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ ∠BAE = ∠QPL … (1)

Similarly, it can be proved that ΔAEC ~ ΔPLR and

∠CAE = ∠RPL … (2)

Adding equation (1) and (2), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒ ∠CAB = ∠RPQ … (3)

In ΔABC and ΔPQR,

$\frac{AB}{PQ}=\frac{AC}{PR}$ (Given)

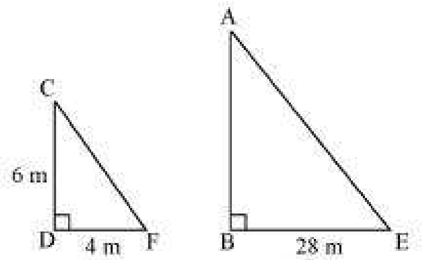
∠CAB = ∠RPQ [Using equation (3)]

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 15:

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

$\therefore \Delta ABE \sim \Delta CDF$ (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6\text{ m}} = \frac{28}{4}$$

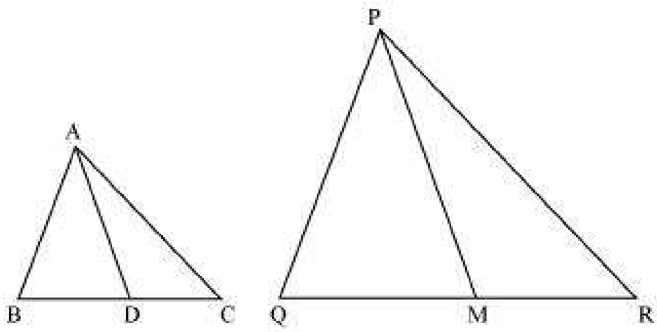
$$\Rightarrow AB = 42\text{ m}$$

Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ prove tha t $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer:



It is given that ΔABC ~ ΔPQR

We know that the corresponding sides of similar triangles are in proportion.

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ ... (1)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ... (2)

Since AD and PM are medians, they will divide their opposite sides.

$\therefore BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ ... (3)

From equations (1) and (3), we obtain

$\frac{AB}{PQ} = \frac{BD}{QM}$ ... (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using equation (4)]

∴ ΔABD ~ ΔPQM (By SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Exercise 6.4

Question 1:

Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Answer:

It is given that $\Delta ABC \sim \Delta DEF$.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$EF = 15.4 \text{ cm},$

$ar(\Delta ABC) = 64 \text{ cm}^2,$

$ar(\Delta DEF) = 121 \text{ cm}^2$

$$\therefore \frac{ar(ABC)}{ar(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64 \text{ cm}^2}{121 \text{ cm}^2}\right) = \frac{BC^2}{(15.4 \text{ cm})^2}$$

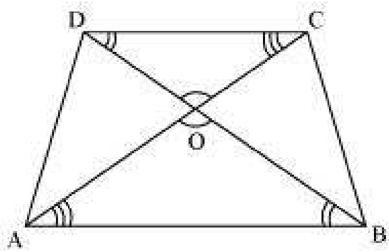
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right)\text{cm}$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right)\text{cm} = (8 \times 1.4)\text{cm} = 11.2\text{cm}$$

Question 2:

Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Answer:



Since AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

∴ ΔAOB ~ ΔCOD (By AAA similarity criterion)

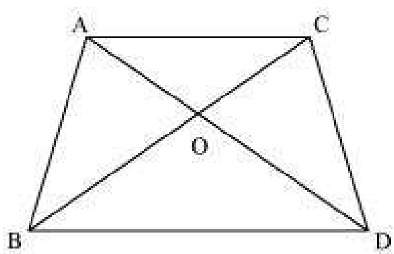
$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$$

Since $AB = 2\,CD$,

$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{2\,CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

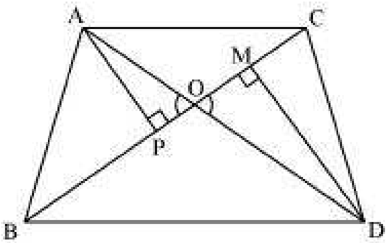
Question 3:

In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AO}{DO}$



Answer:

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}.$$

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO ~ ΔDMO (By AA similarity criterion)

$$\therefore \frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.

Answer:

Let us assume two similar triangles as ΔABC ~ ΔPQR.

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \qquad (1)$$

Given that, ar $(\Delta ABC) =$ ar (ΔPQR)

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

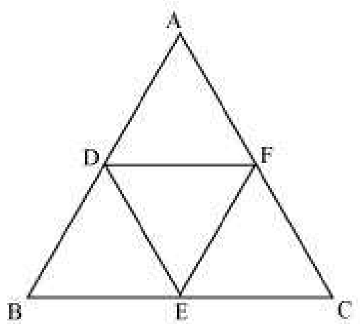
$\Rightarrow AB = PQ, BC = QR,$ and $AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Answer:



D and E are the mid-points of ΔABC.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2}AC$

In ΔBED and ΔBCA,

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

$\therefore \Delta BED \sim \Delta BCA$ (AAA similarity criterion)

$$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BED) = \frac{1}{4}ar(\Delta BCA)$$

Similarly, $ar(\Delta CFE) = \frac{1}{4}ar(CBA)$ and $ar(\Delta ADF) = \frac{1}{4}ar(\Delta ABC)$

Also, $ar(\Delta DEF) = ar(\Delta ABC) - \left[ar(\Delta BED) + ar(\Delta CFE) + ar(\Delta ADF)\right]$

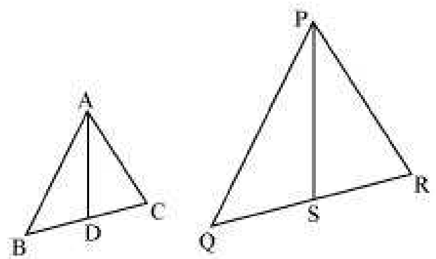
$$\Rightarrow ar(\Delta DEF) = ar(\Delta ABC) - \frac{3}{4}ar(\Delta ABC) = \frac{1}{4}ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer:



Let us assume two similar triangles as ΔABC ~ ΔPQR. Let AD and PS be the medians of these triangles.

∵ ΔABC ~ ΔPQR

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ...(1)

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ... (2)

Since AD and PS are medians,

∴ BD = DC = $\frac{BC}{2}$

And, QS = SR = $\frac{QR}{2}$

Equation (1) becomes

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ ... (3)

In ΔABD and ΔPQS,

∠B = ∠Q [Using equation (2)]

And, $\frac{AB}{PQ} = \frac{BD}{QS}$ [Using equation (3)]

∴ ΔABD ~ ΔPQS (SAS similarity criterion)

Therefore, it can be said that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$ ... (4)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

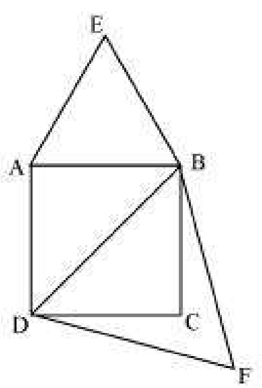
And hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer:



Let ABCD be a square of side a.

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF.

Side of an equilateral triangle, ΔABE, described on one of its sides = *a*

Side of an equilateral triangle, ΔDBF, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \Delta\,\text{ABE}}{\text{Area of } \Delta\,\text{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Question 8:

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

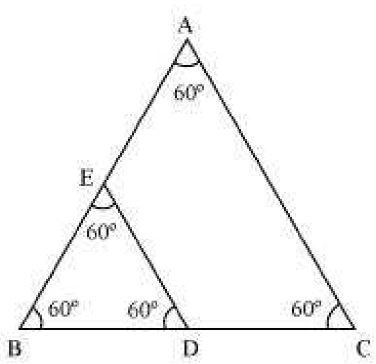
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Answer:



We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of ΔABC = x

Therefore, side of $\Delta\text{BDE} = \frac{x}{2}$

$$\therefore \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Question 9:

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Answer:

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Exercise 6.5

Question 1:

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer:

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

49 + 576 = 625

Or, $7^2 + 24^2 = 25^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

However, $9 + 36 \neq 64$

Or, $3^2 + 6^2 \neq 8^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii)Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

However, $2500 + 6400 \neq 10000$

Or, $50^2 + 80^2 \neq 100^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv)Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, 144 +25 = 169

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

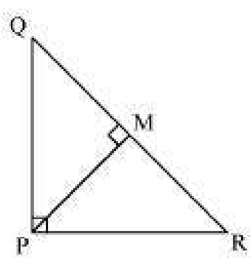
We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Question 2:

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2 = QM \times MR$.

Answer:



Let $\angle MPR = x$

In ΔMPR,

$\angle MRP = 180^\circ - 90^\circ - x$

$\angle MRP = 90^\circ - x$

Similarly, in ΔMPQ,

$\angle MPQ = 90^\circ - \angle MPR$

$\quad\quad\quad = 90^\circ - x$

$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$

$\angle MQP = x$

In Δ QMP and ΔPMR,

$\angle MPQ = \angle MRP$

$\angle PMQ = \angle RMP$

$\angle MQP = \angle MPR$

$\therefore \Delta QMP \sim \Delta PMR \quad\quad$ (By AAA similarity criterion)

$$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$$

$\Rightarrow PM^2 = QM \times MR$

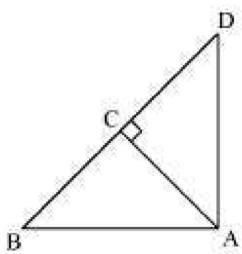
Question 3:

In the following figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC \times BD$

(ii) $AC^2 = BC \times DC$

(iii) $AD^2 = BD \times CD$



Answer:

(i) In ΔADB and ΔCAB

$\angle DAB = \angle ACB$ (Each 90°)

$\angle ABD = \angle CBA$ (Common angle)

$\therefore \Delta ADB \sim \Delta CAB$ (AA similarity criterion)

$\Rightarrow \frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$

(ii) Let $\angle CAB = x$

In ΔCBA,

$\angle CBA = 180^\circ - 90^\circ - x$

$\angle CBA = 90^\circ - x$

Similarly, in ΔCAD,

$\angle CAD = 90^\circ - \angle CAB$

$= 90^\circ - x$

$\angle CDA = 180^\circ - 90^\circ - (90^\circ - x)$

$\angle CDA = x$

In Δ CBA and Δ CAD,

$\angle CBA = \angle CAD$

$\angle CAB = \angle CDA$

$\angle ACB = \angle DCA$ (Each 90°)

$\therefore \Delta CBA \sim \Delta CAD$ (By AAA rule)

$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = DC \times BC$

(iii) In ΔDCA and ΔDAB,

$\angle DCA = \angle DAB$ (Each 90°)

$\angle CDA = \angle ADB$ (Common angle)

$\therefore \Delta DCA \sim \Delta DAB$ (AA similarity criterion)

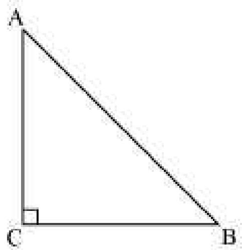
$\Rightarrow \frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

Question 4:

ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2\ AC^2$.

Answer:



Given that ΔABC is an isosceles triangle.

∴ AC = CB

Applying Pythagoras theorem in ΔABC (i.e., right-angled at point C), we obtain

$AC^2 + CB^2 = AB^2$

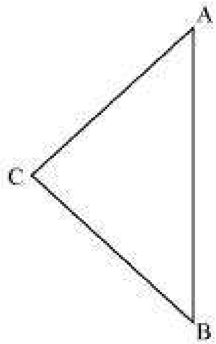
$\Rightarrow AC^2 + AC^2 = AB^2 \qquad (AC = CB)$

$\Rightarrow 2AC^2 = AB^2$

Question 5:

ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

Answer:



Given that,

$AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = AC^2 + BC^2 \quad (\text{As } AC = BC)$

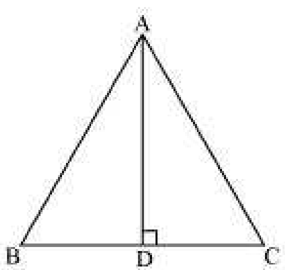
The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Question 6:

ABC is an equilateral triangle of side 2*a*. Find each of its altitudes.

Answer:



Let AD be the altitude in the given equilateral triangle, ΔABC.

We know that altitude bisects the opposite side.

∴ BD = DC = *a*

In ΔADB,

$\angle ADB = 90°$

Applying pythagoras theorem, we obtain

$$AD^2 + DB^2 = AB^2$$
$$\Rightarrow AD^2 + a^2 = (2a)^2$$
$$\Rightarrow AD^2 + a^2 = 4a^2$$
$$\Rightarrow AD^2 = 3a^2$$
$$\Rightarrow AD = a\sqrt{3}$$

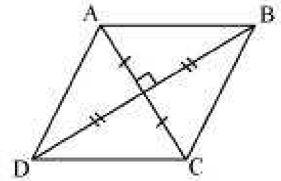
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Question 7:

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer:



In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$AB^2 = AO^2 + OB^2$ ... (1)

$BC^2 = BO^2 + OC^2$ ... (2)

$CD^2 = CO^2 + OD^2$ ... (3)

$AD^2 = AO^2 + OD^2$ ... (4)

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

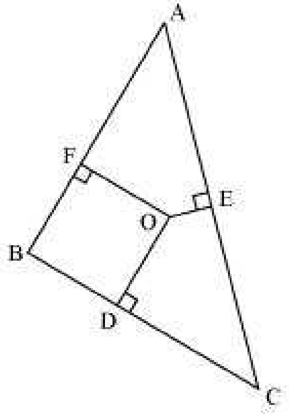
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

Question 8:

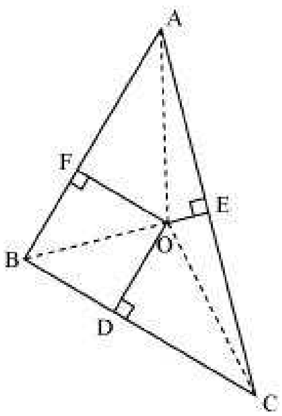
In the following figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that



(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer:



(i) Applying Pythagoras theorem in ΔAOF, we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in ΔBOD,

$$OB^2 = OD^2 + BD^2$$

Similarly, in ΔCOE,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

(ii) From the above result,

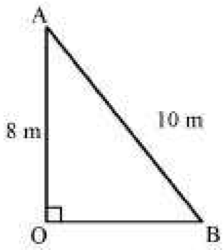
$$AF^2 + BD^2 + EC^2 = \left(OA^2 - OE^2\right) + \left(OC^2 - OD^2\right) + \left(OB^2 - OF^2\right)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer:



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$$AB^2 = OA^2 + BO^2$$

$$(10\text{ m})^2 = (8\text{ m})^2 + OB^2$$

$$100\text{ m}^2 = 64\text{ m}^2 + OB^2$$

$$OB^2 = 36\text{ m}^2$$

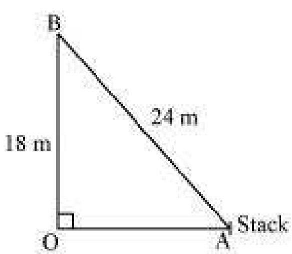
$$OB = 6\text{ m}$$

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer:



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

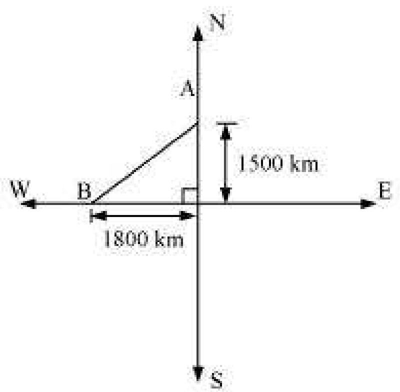
$$AB^2 = OB^2 + OA^2$$
$$(24\text{ m})^2 = (18\text{ m})^2 + OA^2$$
$$OA^2 = (576-324)\text{m}^2 = 252\text{ m}^2$$
$$OA = \sqrt{252}\text{ m} = \sqrt{6\times6\times7}\text{ m} = 6\sqrt{7}\text{ m}$$

Therefore, the distance from the base is $6\sqrt{7}$ m.

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer:



Distance travelled by the plane flying towards north in $1\frac{1}{2}$ hrs

$$= 1{,}000 \times 1\frac{1}{2} = 1{,}500\text{ km}$$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}$ hrs

$$= 1{,}200 \times 1\frac{1}{2} = 1{,}800\text{ km}$$

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

Distance between these planes after $1\frac{1}{2}$ hrs, AB = $\sqrt{OA^2 + OB^2}$

$$= \left(\sqrt{(1,500)^2 + (1,800)^2}\right) \text{km} = \left(\sqrt{2250000 + 3240000}\right) \text{km}$$

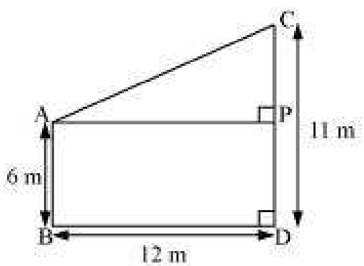
$$= \left(\sqrt{5490000}\right) \text{km} = \left(\sqrt{9 \times 610000}\right) \text{km} = 300\sqrt{61} \text{ km}$$

Therefore, the distance between these planes will be $300\sqrt{61}$ km after $1\frac{1}{2}$ hrs.

Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer:



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, CP = 11 − 6 = 5 m

From the figure, it can be observed that AP = 12m

Applying Pythagoras theorem for ΔAPC, we obtain

$AP^2 + PC^2 = AC^2$

$(12\text{ m})^2 + (5\text{ m})^2 = AC^2$

$AC^2 = (144 + 25)\text{m}^2 = 169\text{ m}^2$

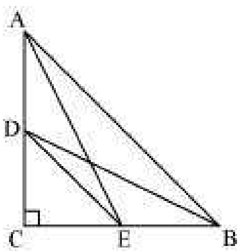
$AC = 13\text{ m}$

Therefore, the distance between their tops is 13 m.

Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Answer:



Applying Pythagoras theorem in ΔACE, we obtain

$AC^2 + CE^2 = AE^2 \quad \ldots (1)$

Applying Pythagoras theorem in ΔBCD, we obtain

$BC^2 + CD^2 = BD^2 \quad \ldots (2)$

Using equation (1) and equation (2), we obtain

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \ldots (3)$

Applying Pythagoras theorem in ΔCDE, we obtain

$DE^2 = CD^2 + CE^2$

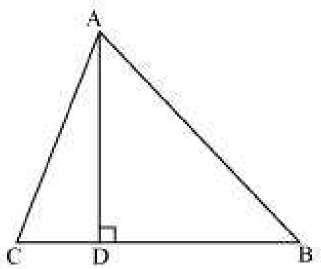
Applying Pythagoras theorem in ΔABC, we obtain

$AB^2 = AC^2 + CB^2$

Putting the values in equation (3), we obtain

$DE^2 + AB^2 = AE^2 + BD^2$

The perpendicular from A on side BC of a ΔABC intersect BC at D such that DB = 3 CD. Prove that 2 AB^2 = 2 AC^2 + BC^2



Answer:

Applying Pythagoras theorem for ΔACD, we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \qquad \ldots (1)$$

Applying Pythagoras theorem in ΔABD, we obtain

$$AB^2 = AD^2 + DB^2$$

$$AD^2 = AB^2 - DB^2 \qquad \ldots (2)$$

From equation (1) and equation (2), we obtain

$$AC^2 - DC^2 = AB^2 - DB^2 \qquad \ldots (3)$$

It is given that 3DC = DB

$$\therefore DC = \frac{BC}{4} \text{ and } DB = \frac{3BC}{4}$$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

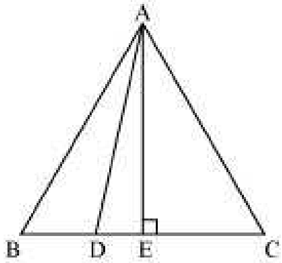
$$16AB^2 - 16AC^2 = 8BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

Question 15:

In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that 9 AD^2 = 7 AB^2.

Answer:



Let the side of the equilateral triangle be a, and AE be the altitude of ΔABC.

$\therefore$ BE = EC = $\frac{BC}{2} = \frac{a}{2}$

And, AE = $\frac{a\sqrt{3}}{2}$

Given that, BD = $\frac{1}{3}$ BC

$\therefore$ BD = $\frac{a}{3}$

DE = BE − BD = $\frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

Applying Pythagoras theorem in ΔADE, we obtain

$AD^2 = AE^2 + DE^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

$$= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right)$$

$$= \frac{28a^2}{36}$$

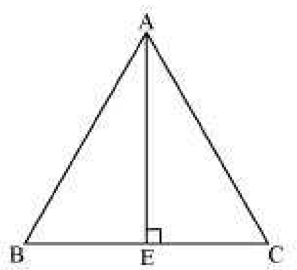
$$= \frac{7}{9}AB^2$$

$\Rightarrow 9\ AD^2 = 7\ AB^2$

Question 16:

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer:



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

$\therefore$ BE = EC = $\frac{BC}{2} = \frac{a}{2}$

Applying Pythagoras theorem in ΔABE, we obtain

$AB^2 = AE^2 + BE^2$

$$a^2 = \mathrm{AE}^2 + \left(\frac{a}{2}\right)^2$$

$$\mathrm{AE}^2 = a^2 - \frac{a^2}{4}$$

$$\mathrm{AE}^2 = \frac{3a^2}{4}$$

$4\mathrm{AE}^2 = 3a^2$

$\Rightarrow$ 4 × (Square of altitude) = 3 × (Square of one side)

Question 17:

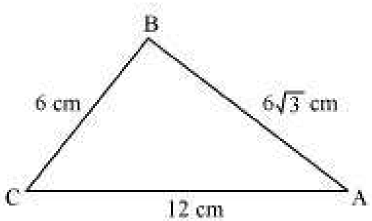
Tick the correct answer and justify: In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Answer:



Given that, AB = $6\sqrt{3}$ cm, AC = 12 cm, and BC = 6 cm

It can be observed that

$\mathrm{AB}^2 = 108$

$\mathrm{AC}^2 = 144$

And, $\mathrm{BC}^2 = 36$

$\mathrm{AB}^2 + \mathrm{BC}^2 = \mathrm{AC}^2$

The given triangle, ΔABC, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.

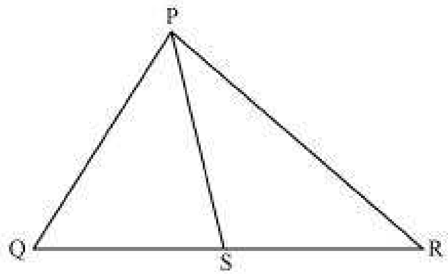
$\therefore \angle B = 90°$

Hence, the correct answer is (C).

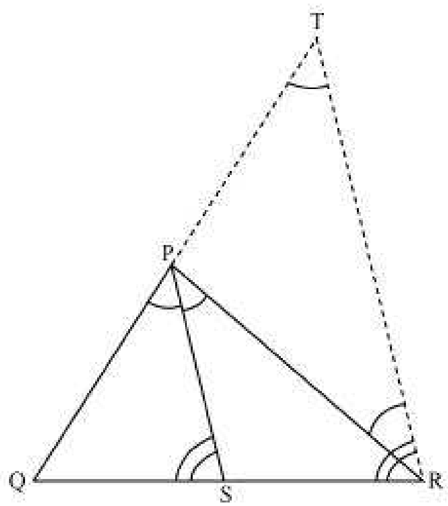
Exercise 6.6

Question 1:

In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that $\frac{QS}{SR}=\frac{PQ}{PR}$.



Answer:



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of ∠QPR.

∠QPS = ∠SPR … (1)

By construction,

∠SPR = ∠PRT (As PS || TR) … (2)

∠QPS = ∠QTR (As PS || TR) … (3)

Using these equations, we obtain

∠PRT = ∠QTR

∴ PT = PR

By construction,

PS || TR

By using basic proportionality theorem for ΔQTR,

$$\frac{QS}{SR} = \frac{QP}{PT}$$

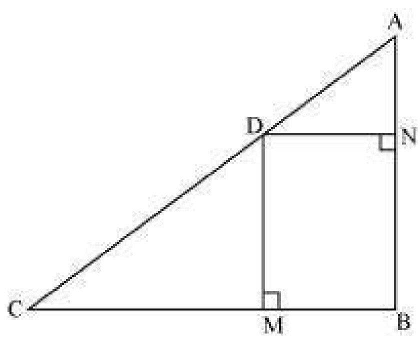
$$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR} \qquad (PT = TR)$$

Question 2:

In the given figure, D is a point on hypotenuse AC of ΔABC, DM ⊥ BC and DN ⊥ AB, Prove that:

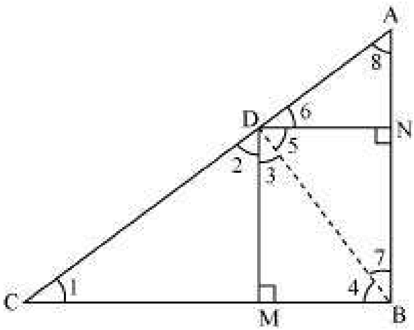
(i) $DM^2 = DN.MC$

(ii) $DN^2 = DM.AN$



Answer:

(i)Let us join DB.



We have, DN || CB, DM || AB, and ∠B = 90°

∴ DMBN is a rectangle.

∴ DN = MB and DM = NB

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

∴ ∠CDB = 90°

⇒ ∠2 + ∠3 = 90° … (1)

In ΔCDM,

∠1 + ∠2 + ∠DMC = 180°

⇒ ∠1 + ∠2 = 90° … (2)

In ΔDMB,

∠3 + ∠DMB + ∠4 = 180°

⇒ ∠3 + ∠4 = 90° … (3)

From equation (1) and (2), we obtain

∠1 = ∠3

From equation (1) and (3), we obtain

∠2 = ∠4

In ΔDCM and ΔBDM,

∠1 = ∠3 (Proved above)

∠2 = ∠4 (Proved above)

∴ ΔDCM ~ ΔBDM (AA similarity criterion)

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$

$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \qquad (BM = DN)$$

$\Rightarrow DM^2 = DN \times MC$

(ii) In right triangle DBN,

$\angle 5 + \angle 7 = 90°$ … (4)

In right triangle DAN,

$\angle 6 + \angle 8 = 90°$ … (5)

D is the foot of the perpendicular drawn from B to AC.

$\therefore \angle ADB = 90°$

$\Rightarrow \angle 5 + \angle 6 = 90°$ … (6)

From equation (4) and (6), we obtain

$\angle 6 = \angle 7$

From equation (5) and (6), we obtain

$\angle 8 = \angle 5$

In ΔDNA and ΔBND,

$\angle 6 = \angle 7$ (Proved above)

$\angle 8 = \angle 5$ (Proved above)

$\therefore \Delta DNA \sim \Delta BND$ (AA similarity criterion)

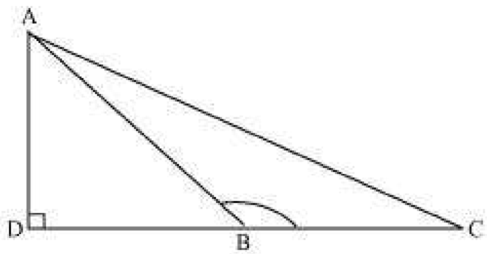
$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

$\Rightarrow DN^2 = AN \times NB$

$\Rightarrow DN^2 = AN \times DM$ (As NB = DM)

Question 3:

In the given figure, ABC is a triangle in which $\angle ABC > 90°$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer:

Applying Pythagoras theorem in ΔADB, we obtain

$AB^2 = AD^2 + DB^2$ … (1)

Applying Pythagoras theorem in ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

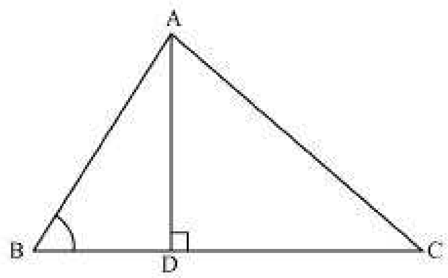
$AC^2 = AD^2 + (DB + BC)^2$

$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$AC^2 = AB^2 + BC^2 + 2DB \times BC$ [Using equation (1)]

Question 4:

In the given figure, ABC is a triangle in which ∠ABC < 90° and AD ⊥ BC. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Answer:

Applying Pythagoras theorem in ΔADB, we obtain

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 = AB^2 - DB^2$ … (1)

Applying Pythagoras theorem in ΔADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 - BD^2 + DC^2 = AC^2$ [Using equation (1)]

$AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$

$= AB^2 + BC^2 - 2BC \times BD$

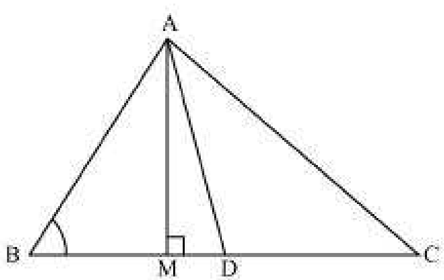
Question 5:

In the given figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that:

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Answer:

(i) Applying Pythagoras theorem in ΔAMD, we obtain

$AM^2 + MD^2 = AD^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$

$AM^2 + (MD + DC)^2 = AC^2$

$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$AD^2 + DC^2 + 2MD.DC = AC^2$ [Using equation (1)]

Using the result, $DC = \frac{BC}{2}$, we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in ΔABM, we obtain

$AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii)Applying Pythagoras theorem in ΔABM, we obtain

$AM^2 + MB^2 = AB^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$ … (2)

Adding equations (1) and (2), we obtain

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$

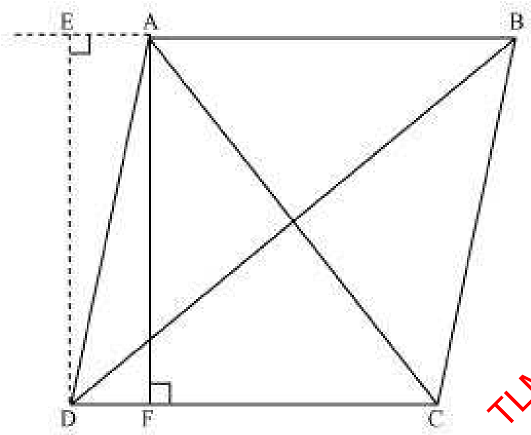
$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$

Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer:



Let ABCD be a parallelogram.

Let us draw perpendicular DE on extended side AB, and AF on side DC.

Applying Pythagoras theorem in ΔDEA, we obtain

$DE^2 + EA^2 = DA^2$ … (*i*)

Applying Pythagoras theorem in ΔDEB, we obtain

$DE^2 + EB^2 = DB^2$

$DE^2 + (EA + AB)^2 = DB^2$

$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$

$DA^2 + AB^2 + 2EA \times AB = DB^2$ … (*ii*)

Applying Pythagoras theorem in ΔADF, we obtain

$AD^2 = AF^2 + FD^2$

Applying Pythagoras theorem in ΔAFC, we obtain

$AC^2 = AF^2 + FC^2$

$= AF^2 + (DC - FD)^2$

$= AF^2 + DC^2 + FD^2 - 2DC \times FD$

$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$

$AC^2 = AD^2 + DC^2 - 2DC \times FD$ … (*iii*)

Since ABCD is a parallelogram,

AB = CD … (*iv*)

And, BC = AD … (*v*)

In ΔDEA and ΔADF,

∠DEA = ∠AFD (Both 90°)

∠EAD = ∠ADF (EA || DF)

AD = AD (Common)

∴ ΔEAD ≅ ΔFDA (AAS congruence criterion)

⇒ EA = DF … (vi)

Adding equations (*i*) and (*iii*), we obtain

$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$

$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$

$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$

[Using equations (*iv*) and (*vi*)]

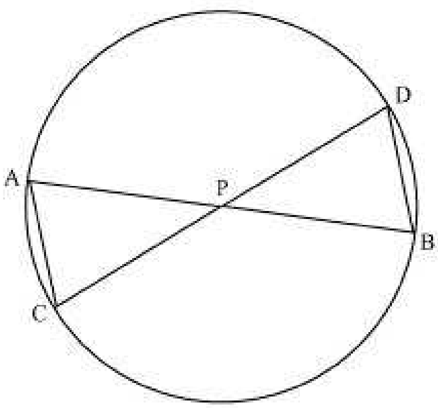
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Question 7:

In the given figure, two chords AB and CD intersect each other at the point P. prove that:

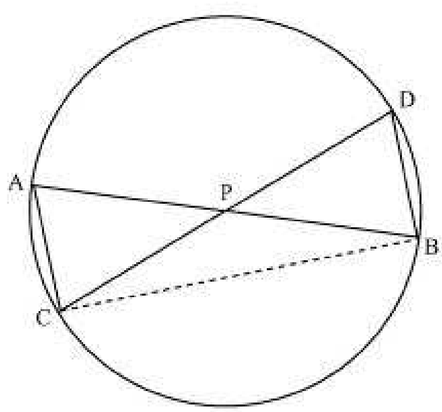
(i) ΔAPC ~ ΔDPB

(ii) AP.BP = CP.DP



Answer:

Let us join CB.



(i) In ΔAPC and ΔDPB,

∠APC = ∠DPB (Vertically opposite angles)

∠CAP = ∠BDP (Angles in the same segment for chord CB)

ΔAPC ~ ΔDPB (By AA similarity criterion)

(ii) We have already proved that

ΔAPC ~ ΔDPB

We know that the corresponding sides of similar triangles are proportional.

$$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$$

$$\Rightarrow \frac{AP}{DP} = \frac{PC}{PB}$$

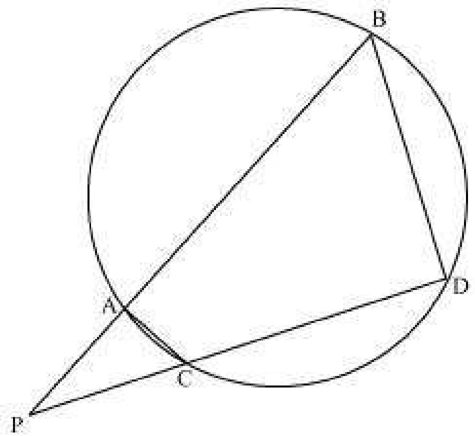
∴ AP. PB = PC. DP

Question 8:

In the given figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) ΔPAC ~ ΔPDB

(ii) PA.PB = PC.PD



Answer:

(i) In ΔPAC and ΔPDB,

∠P = ∠P (Common)

∠PAC = ∠PDB (Exterior angle of a cyclic quadrilateral is ∠PCA = ∠PBD equal to the opposite interior angle)

∴ ΔPAC ~ ΔPDB

(ii)We know that the corresponding sides of similar triangles are proportional.

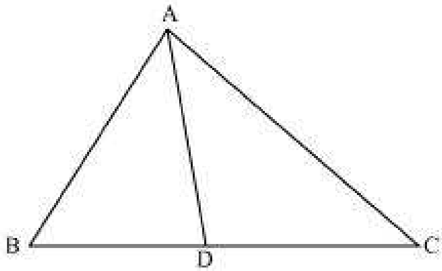
$$\therefore \frac{PA}{PD}=\frac{AC}{DB}=\frac{PC}{PB}$$

$$\Rightarrow \frac{PA}{PD}=\frac{PC}{PB}$$

∴ PA.PB = PC.PD

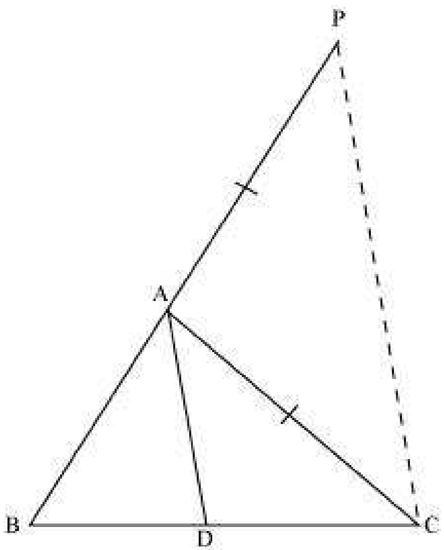
Question 9:

In the given figure, D is a point on side BC of ΔABC such that $\frac{BD}{CD}=\frac{AB}{AC}$. Prove that AD is the bisector of ∠BAC.



Answer:

Let us extend BA to P such that AP = AC. Join PC.



It is given that,

$$\frac{BD}{CD} = \frac{AB}{AC}$$

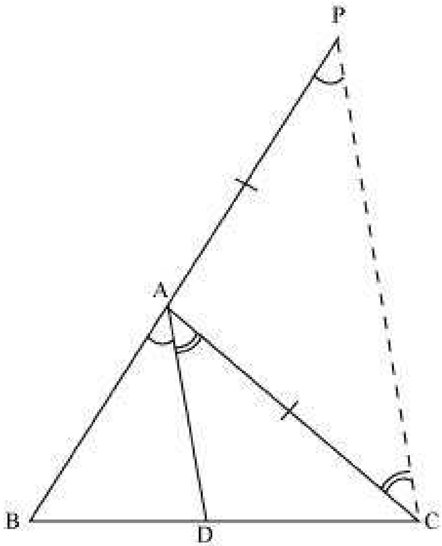
$$\Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$

By using the converse of basic proportionality theorem, we obtain

AD || PC

⇒ ∠BAD = ∠APC (Corresponding angles) … (1)

And, ∠DAC = ∠ACP (Alternate interior angles) … (2)



By construction, we have

AP = AC

⇒ ∠APC = ∠ACP … (3)

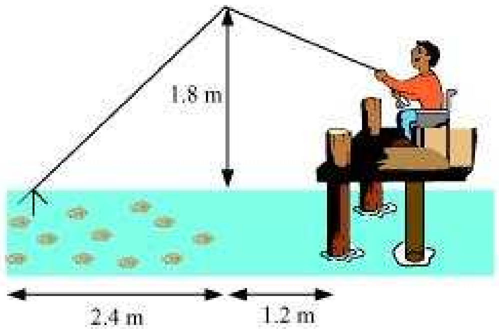
On comparing equations (1), (2), and (3), we obtain

∠BAD = ∠APC

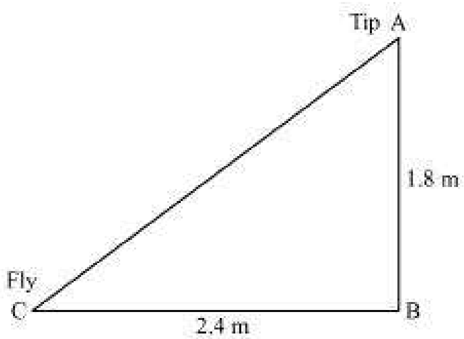
⇒ AD is the bisector of the angle BAC

Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, ho much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Answer:



Let AB be the height of the tip of the fishing rod from the water surface. Let BC be the horizontal distance of the fly from the tip of the fishing rod.

Then, AC is the length of the string.

AC can be found by applying Pythagoras theorem in ΔABC.

$AC^2 = AB^2 + BC^2$

$AB^2 = (1.8\text{ m})^2 + (2.4\text{ m})^2$

$AB^2 = (3.24 + 5.76)\text{ m}^2$

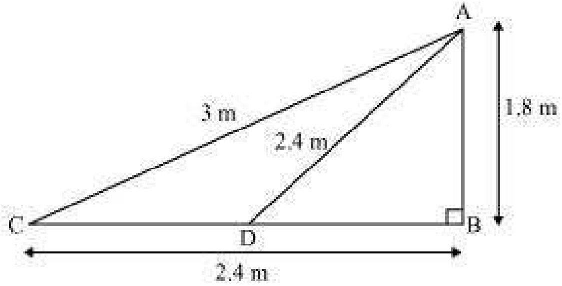
$AB^2 = 9.00\text{ m}^2$

$\Rightarrow AB = \sqrt{9}\text{ m} = 3\text{ m}$

Thus, the length of the string out is 3 m.

She pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = 12 × 5 = 60 cm = 0.6 m



Let the fly be at point D after 12 seconds.

Length of string out after 12 seconds is AD.

AD = AC − String pulled by Nazima in 12 seconds

= (3.00 − 0.6) m

= 2.4 m

In ΔADB,

$AB^2 + BD^2 = AD^2$

$(1.8\text{ m})^2 + BD^2 = (2.4\text{ m})^2$

$BD^2 = (5.76 - 3.24)\text{ m}^2 = 2.52\text{ m}^2$

BD = 1.587 m

Horizontal distance of fly = BD + 1.2 m

= (1.587 + 1.2) m

= 2.787 m

= 2.79 m

Exercise 8.1

Question 1:

In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A

(ii) sin C, cos C

Answer:

Applying Pythagoras theorem for ΔABC, we obtain

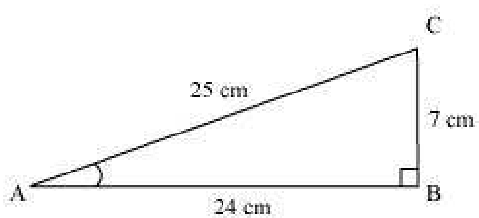
$AC^2 = AB^2 + BC^2$

$= (24\text{ cm})^2 + (7\text{ cm})^2$

$= (576 + 49)\text{ cm}^2$

$= 625\text{ cm}^2$

$\therefore AC = \sqrt{625}\text{ cm} = 25\text{ cm}$

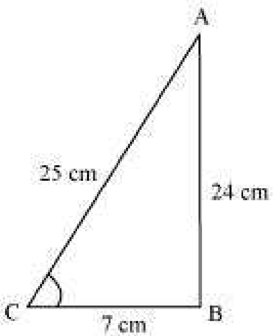


(i) $\sin A = \dfrac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

$= \dfrac{7}{25}$

$\cos A = \dfrac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{24}{25}$

(ii)



sin C = $\frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC}$

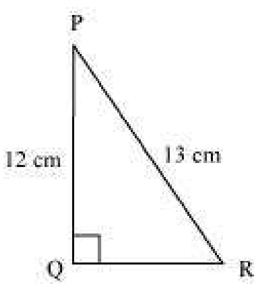
$= \frac{24}{25}$

cos C = $\frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC}$

$= \frac{7}{25}$

Question 2:

In the given figure find tan P − cot R



Answer:

Applying Pythagoras theorem for ΔPQR, we obtain

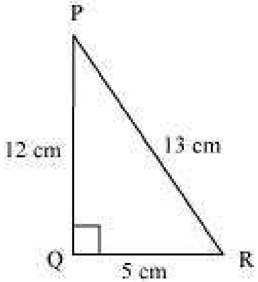
$PR^2 = PQ^2 + QR^2$

$(13 \text{ cm})^2 = (12 \text{ cm})^2 + QR^2$

$169\ cm^2 = 144\ cm^2 + QR^2$

$25\ cm^2 = QR^2$

QR = 5 cm



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

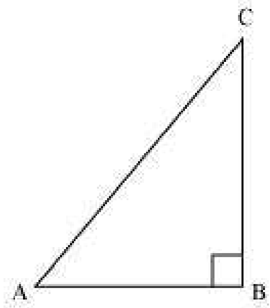
tan P − cot R = $\frac{5}{12} - \frac{5}{12} = 0$

Question 3:

If sin A = $\frac{3}{4}$, calculate cos A and tan A.

Answer:

Let ΔABC be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be $3k$. Therefore, AC will be $4k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

AB = $\sqrt{7}k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

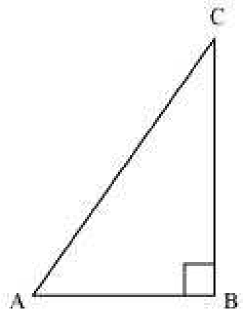
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Question 4:

Given 15 cot A = 8. Find sin A and sec A

Answer:

Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

It is given that,

cot A = $\frac{8}{15}$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be $8k$.Therefore, BC will be $15k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

$AC = 17k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

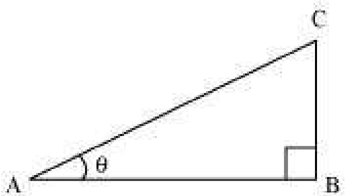
$$= \frac{AC}{AB} = \frac{17}{8}$$

Question 5:

Given sec θ = $\frac{13}{12}$, calculate all other trigonometric ratios.

Answer:

Consider a right-angle triangle ΔABC, right-angled at point B.



$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle\theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is 13*k*, AB will be 12*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

BC = 5k

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{\text{AB}}{\text{AC}} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \frac{\text{BC}}{\text{AB}} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{\text{AB}}{\text{BC}} = \frac{12k}{5k} = \frac{12}{5}$$

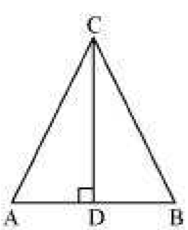
$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle\theta} = \frac{\text{AC}}{\text{BC}} = \frac{13k}{5k} = \frac{13}{5}$$

Question 6:

If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Answer:

Let us consider a triangle ABC in which CD ⊥ AB.

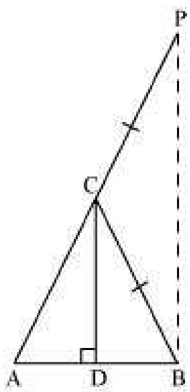


It is given that

cos A = cos B

$$\Rightarrow \frac{\text{AD}}{\text{AC}} = \frac{\text{BD}}{\text{BC}} \quad \ldots (1)$$

We have to prove $\angle A = \angle B$. To prove this, let us extend AC to P such that BC = CP.



From equation (1), we obtain

$$\frac{AD}{BD} = \frac{AC}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{CP} \qquad \left(\text{By construction, we have } BC = CP\right) \qquad \dots (2)$$

By using the converse of B.P.T,

CD||BP

$\Rightarrow \angle ACD = \angle CPB$ (Corresponding angles) … (3)

And, $\angle BCD = \angle CBP$ (Alternate interior angles) … (4)

By construction, we have BC = CP.

$\therefore\ \angle CBP = \angle CPB$ (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

$\angle ACD = \angle BCD$ … (6)

In ΔCAD and ΔCBD,

$\angle ACD = \angle BCD$ [Using equation (6)]

$\angle CDA = \angle CDB$ [Both 90°]

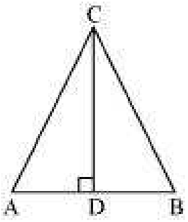
Therefore, the remaining angles should be equal.

$\therefore \angle CAD = \angle CBD$

$\Rightarrow \angle A = \angle B$

Alternatively,

Let us consider a triangle ABC in which CD $\perp$ AB.



It is given that,

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

Let $\frac{AD}{BD} = \frac{AC}{BC} = k$

$\Rightarrow$ AD = k BD … (1)

And, AC = k BC … (2)

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$CD^2 = AC^2 - AD^2$ … (3)

And, $CD^2 = BC^2 - BD^2$ … (4)

From equations (3) and (4), we obtain

$AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (k\,BC)^2 - (k\,BD)^2 = BC^2 - BD^2$

$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1$

$\Rightarrow k = 1$

Putting this value in equation (2), we obtain

AC = BC

$\Rightarrow \angle A = \angle B$(Angles opposite to equal sides of a triangle)

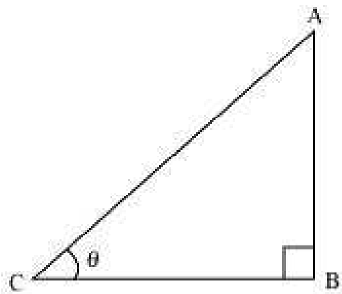
Question 7:

If $\cot\theta = \frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Answer:

Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

If BC is $7k$, then AB will be $8k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

AC = $\sqrt{113}k$

$$\sin\theta = \frac{\text{Side opposite to }\angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to }\angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Question 8:

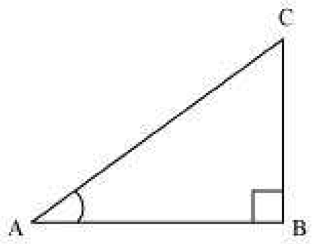
If 3 cot A = 4, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer:

It is given that 3cot A = 4

Or, cot A = $\frac{4}{3}$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is $4k$, then BC will be $3k$, where k is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

AC = $5k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4k}{5k} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3k}{5k} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}=\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$=\frac{\frac{7}{16}}{\frac{25}{16}}=\frac{7}{25}$$

$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2$

$$=\frac{16}{25}-\frac{9}{25}=\frac{7}{25}$$

$$\therefore \frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$$

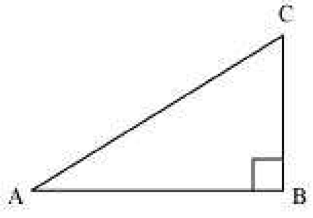
Question 9:

In ΔABC, right angled at B. If $\tan A=\frac{1}{\sqrt{3}}$, find the value of

(i) sin A cos C + cos A sin C

(ii) cos A cos C − sin A sin C

Answer:



$\tan A = \frac{1}{\sqrt{3}}$

$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

If BC is k, then AB will be $\sqrt{3}k$, where k is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}k\right)^2 + (k)^2$

$= 3k^2 + k^2 = 4k^2$

$\therefore AC = 2k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) cos A cos C − sin A sin C

$$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}=0$$

Question 10:

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

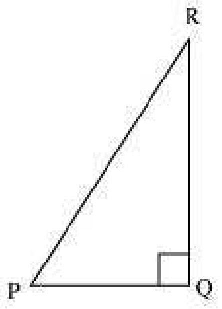
Answer:

Given that, PR + QR = 25

PQ = 5

Let PR be x.

Therefore, QR = 25 − x



Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

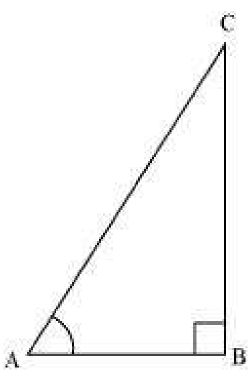
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

(v) sin θ = $\frac{4}{3}$, for some angle θ

Answer:

(i) Consider a ΔABC, right-angled at B.



$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{12}{5}$$

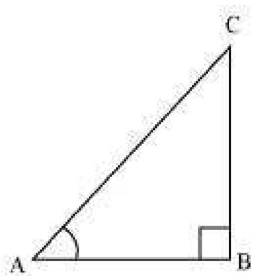
But $\frac{12}{5} > 1$

$\therefore$ tan A > 1

So, tan A < 1 is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be $12k$, AB will be $5k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

$BC = 10.9k$

It can be observed that for given two sides AC = $12k$ and AB = $5k$,

BC should be such that,

AC − AB < BC < AC + AB

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17\ k$

However, BC = 10.9k. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of $\angle$A.

Hence, the given statement is false.

(v) $\sin\theta = \frac{4}{3}$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible.

Hence, the given statement is false

Exercise 8.2

Question 1:

Evaluate the following

(i) sin60° cos30° + sin30° cos 60°

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Answer:

(i) sin60° cos30° + sin30° cos 60°

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$=\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}=\frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$=\frac{\sqrt{3}}{\sqrt{2}\left(2+2\sqrt{3}\right)}=\frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$=\frac{\sqrt{3}\left(2\sqrt{6}-2\sqrt{2}\right)}{\left(2\sqrt{6}+2\sqrt{2}\right)\left(2\sqrt{6}-2\sqrt{2}\right)}$$

$$=\frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(2\sqrt{6}\right)^2-\left(2\sqrt{2}\right)^2}=\frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{24-8}=\frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{16}$$

$$=\frac{\sqrt{18}-\sqrt{6}}{8}=\frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30°+\tan 45°-\operatorname{cosec}60°}{\sec 30°+\cos 60°+\cot 45°}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)}$$

$$= \frac{\left(3\sqrt{3}-4\right)\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)\left(3\sqrt{3}-4\right)} = \frac{\left(3\sqrt{3}-4\right)^2}{\left(3\sqrt{3}\right)^2-(4)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4}+\frac{3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{67}{12}$$

Question 2:

Choose the correct option and justify your choice.

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A). sin60°

(B). cos60°

(C). tan60°

(D). sin30°

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A). tan90°

(B). 1

(C). sin45°

(D). 0

(iii) sin2A = 2sinA is true when A =

(A). 0°

(B). 30°

(C). 45°

(D). 60°

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$

(A). cos60°

(B). sin60°

(C). tan60°

(D). sin30°

Answer:

(i) $\dfrac{2\tan 30°}{1+\tan^2 30°}$

$$=\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$=\frac{6}{4\sqrt{3}}=\frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60°=\frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\dfrac{1-\tan^2 45°}{1+\tan^2 45°}$

$$=\frac{1-(1)^2}{1+(1)^2}=\frac{1-1}{1+1}=\frac{0}{2}=0$$

Hence, (D) is correct.

(iii)Out of the given alternatives, only A = 0° is correct.

As sin 2A = sin 0° = 0

2 sinA = 2sin 0° = 2(0) = 0

Hence, (A) is correct.

(iv) $\dfrac{2\tan 30°}{1-\tan^2 30°}$

$$=\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$=\sqrt{3}$$

Out of the given alternatives, only tan 60° $=\sqrt{3}$

Hence, (C) is correct.

Question 3:

If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\dfrac{1}{\sqrt{3}}$;

0° < A + B ≤ 90°, A > B find A and B.

Answer:

$\tan(A+B)=\sqrt{3}$

$\Rightarrow \tan(A+B)=\tan 60$

⇒ A + B = 60 … (1)

$\tan(A-B)=\dfrac{1}{\sqrt{3}}$

⇒ tan (A − B) = tan30

⇒ A − B = 30 … (2)

On adding both equations, we obtain

2A = 90

⇒ A = 45

From equation (1), we obtain

45 + B = 60

B = 15

Therefore, ∠A = 45° and ∠B = 15°

Question 4:

State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B

(ii) The value of sinθ increases as θ increases

(iii) The value of cos θ increases as θ increases

(iv) sinθ = cos θ for all values of θ

(v) cot A is not defined for A = 0°

Answer:

(i) sin (A + B) = sin A + sin B

Let A = 30° and B = 60°

sin (A + B) = sin (30° + 60°)

= sin 90°

= 1

sin A + sin B = sin 30° + sin 60°

$$=\frac{1}{2}+\frac{\sqrt{3}}{2}=\frac{1+\sqrt{3}}{2}$$

Clearly, sin (A + B) ≠ sin A + sin B

Hence, the given statement is false.

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90° as

sin 0° = 0

$$\sin 30°=\frac{1}{2}=0.5$$

$$\sin 45°=\frac{1}{\sqrt{2}}=0.707$$

$$\sin 60°=\frac{\sqrt{3}}{2}=0.866$$

sin 90° = 1

Hence, the given statement is true.

(iii) cos 0° = 1

$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$

$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$

$\cos 60° = \frac{1}{2} = 0.5$

cos90° = 0

It can be observed that the value of cos θ does not increase in the interval of 0° < θ < 90°.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As $\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ.

As $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$,

Hence, the given statement is false.

(v) cot A is not defined for A = 0°

As $\cot A = \frac{\cos A}{\sin A}$,

$\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ = undefined

Hence, the given statement is true.

Exercise 8.3

Question 1:

Evaluate

(I) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(III) cos 48° − sin 42°

(IV)cosec 31° − sec 59°

Answer:

(I) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$

$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$

$= \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

(III)cos 48° − sin 42° = cos (90°− 42°) − sin 42°

= sin 42° − sin 42°

= 0

(IV) cosec 31° − sec 59° = cosec (90° − 59°) − sec 59°

= sec 59° − sec 59°

= 0

Question 2:

Show that

(I) tan 48° tan 23° tan 42° tan 67° = 1

(II)cos 38° cos 52° − sin 38° sin 52° = 0

Answer:

(I) tan 48° tan 23° tan 42° tan 67°

= tan (90° − 42°) tan (90° − 67°) tan 42° tan 67°

= cot 42° cot 67° tan 42° tan 67°

= (cot 42° tan 42°) (cot 67° tan 67°)

= (1) (1)

= 1

(II) cos 38° cos 52° − sin 38° sin 52°

= cos (90° − 52°) cos (90°−38°) − sin 38° sin 52°

= sin 52° sin 38° − sin 38° sin 52°

= 0

Question 3:

If tan 2A = cot (A− 18°), where 2A is an acute angle, find the value of A.

Answer:

Given that,

tan 2A = cot (A− 18°)

cot (90° − 2A) = cot (A −18°)

90° − 2A = A− 18°

108° = 3A

A = 36°

Question 4:

If tan A = cot B, prove that A + B = 90°

Answer:

Given that,

tan A = cot B

tan A = tan (90° − B)

A = 90° − B

$A + B = 90°$

Question 5:

If sec 4A = cosec (A− 20°), where 4A is an acute angle, find the value of A.

Answer:

Given that,

$\sec 4A = \operatorname{cosec}(A - 20°)$

$\operatorname{cosec}(90° - 4A) = \operatorname{cosec}(A - 20°)$

$90° - 4A = A - 20°$

$110° = 5A$

$A = 22°$

Question 6:

If A, Band C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Answer:

We know that for a triangle ABC,

$\angle A + \angle B + \angle C = 180°$

$$\angle B + \angle C = 180° - \angle A$$

$$\frac{\angle B + \angle C}{2} = 90° - \frac{\angle A}{2}$$

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$

$$= \cos\left(\frac{A}{2}\right)$$

Question 7:

Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Answer:

$\sin 67° + \cos 75°$

= $\sin(90° - 23°) + \cos(90° - 15°)$

= $\cos 23° + \sin 15°$

Exercise 8.4

Question 1:

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Answer:

We know that,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1+\cot^2 A}$$

$$\sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\sin A = \pm\frac{1}{\sqrt{1+\cot^2 A}}$$

$\sqrt{1+\cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore, $\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \dfrac{\sin A}{\cos A}$

However, $\cot A = \dfrac{\cos A}{\sin A}$

Therefore, $\tan A = \dfrac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Question 2:

Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer:

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$$\sin A = \sqrt{1-\left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Question 3:

Evaluate

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) sin25° cos65° + cos25° sin65°

Answer:

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$= \frac{[\sin(90° - 27°)]^2 + \sin^2 27°}{[\cos(90° - 73°)]^2 + \cos^2 73°}$

$= \frac{[\cos 27°]^2 + \sin^2 27°}{[\sin 73°]^2 + \cos^2 73°}$

$= \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°}$

$= \frac{1}{1}$ (As $\sin^2 A + \cos^2 A = 1$)

= 1

(ii) sin25° cos65° + cos25° sin65°

$= (\sin 25°)\{\cos(90° - 25°)\} + \cos 25° \{\sin(90° - 25°)\}$

$= (\sin 25°)(\sin 25°) + (\cos 25°)(\cos 25°)$

$= \sin^2 25° + \cos^2 25°$

= 1 (As $\sin^2 A + \cos^2 A = 1$)

Question 4:

Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

(A) 0

(B) 1

(C) 2

(D) −1

(iii) (secA + tanA) (1 − sinA) =

(A) secA

(B) sinA

(C) cosecA

(D) cosA

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$

(B) −1

(C) $\cot^2 A$

(D) $\tan^2 A$

Answer:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9(1)$ [As $\sec^2 A - \tan^2 A = 1$]

$= 9$

Hence, alternative (B) is correct.

(ii)

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$=\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$=\frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$=\frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}=2$$

Hence, alternative (C) is correct.

(iii) (secA + tanA) (1 − sinA)

$$=\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$=\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$=\frac{1-\sin^2 A}{\cos A}=\frac{\cos^2 A}{\cos A}$$

= cosA

Hence, alternative (D) is correct.

(iv) $$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\dfrac{\sin^2 A}{\cos^2 A}}{1+\dfrac{\cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.} = (\operatorname{cosec}\theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \text{R.H.S.}$$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$

$$= \text{R.H.S.}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$= \frac{(1+\sin\theta\ \cos\theta)}{(\sin\theta\ \cos\theta)}$$

= secθ cosec θ +

= R.H.S.

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$$\text{L.H.S.} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}} = (\cos A + 1)$$

$$= \frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A}$$

= R.H.S

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$,

L.H.S = $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \frac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

= cosec A + cot A

= R.H.S

(vi) $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

$$\text{L.H.S.} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} \qquad = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} \qquad = \sec A + \tan A$$

$$= \text{R.H.S.}$$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$

$$= \frac{\sin\theta \times \left(1-2\sin^2\theta\right)}{\cos\theta \times \left\{2\left(1-\sin^2\theta\right)-1\right\}}$$

$$= \frac{\sin\theta \times \left(1-2\sin^2\theta\right)}{\cos\theta \times \left(1-2\sin^2\theta\right)}$$

$$= \tan\theta = \text{R.H.S}$$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\text{L.H.S} = (\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2$$
$$= \sin^2 A + \text{cosec}^2 A + 2\sin A\,\text{cosec}A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$
$$= (\sin^2 A + \cos^2 A) + (\text{cosec}^2 A + \sec^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$$
$$= (1) + (1 + \cot^2 A + 1 + \tan^2 A) + (2) + (2)$$
$$= 7 + \tan^2 A + \cot^2 A$$
$$= \text{R.H.S}$$

(ix) $$(\text{cosec}A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$\text{L.H.S} = (\text{cosec}A - \sin A)(\sec A - \cos A)$$
$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$
$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$
$$= \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A}$$
$$= \sin A \cos A$$

$$\text{R.H.S} = \frac{1}{\tan A + \cot A}$$
$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$
$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

Hence, L.H.S = R.H.S

(x) $$\left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$$

$$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\dfrac{\sin^2 A}{\cos^2 A}}{1+\dfrac{\cos^2 A}{\sin^2 A}}=\frac{\dfrac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\dfrac{\sin^2 A+\cos^2 A}{\sin^2 A}}$$

$$=\frac{\dfrac{1}{\cos^2 A}}{\dfrac{1}{\sin^2 A}}=\frac{\sin^2 A}{\cos^2 A}$$

$$=\tan^2 A$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2=\frac{1+\tan^2 A\ -\ 2\tan A}{1+\cot^2 A-2\cot A}$$

$$=\frac{\sec^2 A-\ 2\tan A}{\operatorname{cosec}^2 A-2\cot A}$$

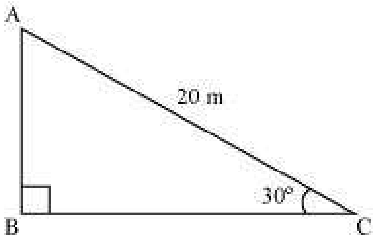
$$=\frac{\dfrac{1}{\cos^2 A}-\dfrac{2\sin A}{\cos A}}{\dfrac{1}{\sin^2 A}-\dfrac{2\cos A}{\sin A}}=\frac{\dfrac{1-2\sin A\cos A}{\cos^2 A}}{\dfrac{1-2\sin A\cos A}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

Exercise 9.1

Question 1:

A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30 °.



Answer:

It can be observed from the figure that AB is the pole.

In ΔABC,

$$\frac{AB}{AC} = \sin 30°$$

$$\frac{AB}{20} = \frac{1}{2}$$

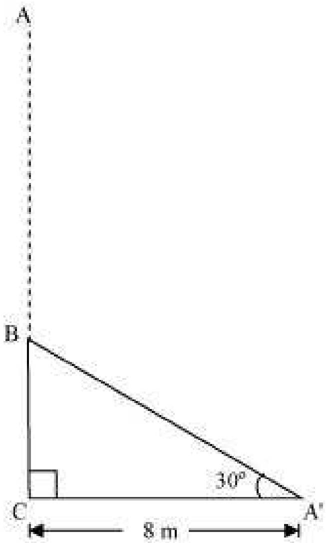
$$AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is 10 m.

Question 2:

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30 ° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Answer:



Let AC was the original tree. Due to storm, it was broken into two parts. The broken part $A'B$ is making 30° with the ground.

In $\Delta A'BC$,

$$\frac{BC}{A'C} = \tan 30°$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \left(\frac{8}{\sqrt{3}}\right) m$$

$$\frac{A'C}{A'B} = \cos 30°$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \left(\frac{16}{\sqrt{3}}\right) m$$

Height of tree = $A'B$ + BC

$$= \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) \text{m} = \frac{24}{\sqrt{3}} \text{ m}$$
$$= 8\sqrt{3} \text{ m}$$

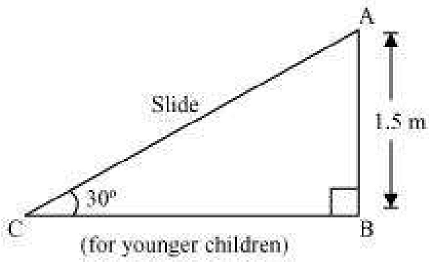
Hence, the height of the tree is $8\sqrt{3}$ m

Question 3:

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30 ° to the ground, where as for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60 ° to the ground. What should be the length of the slide in each case?

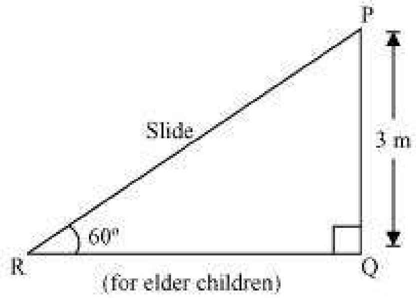
Answer:

It can be observed that AC and PR are the slides for younger and elder children respectively.



In ΔABC,

$$\frac{AB}{AC} = \sin 30°$$
$$\frac{1.5}{AC} = \frac{1}{2}$$
$$AC = 3 \text{ m}$$



In ΔPQR,

$$\frac{PQ}{PR} = \sin 60$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

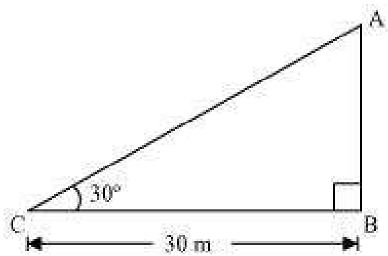
$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}$ m

Question 4:

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.

Answer:



Let AB be the tower and the angle of elevation from point C (on ground) is 30°.

In ΔABC,

$$\frac{AB}{BC} = \tan 30°$$

$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$

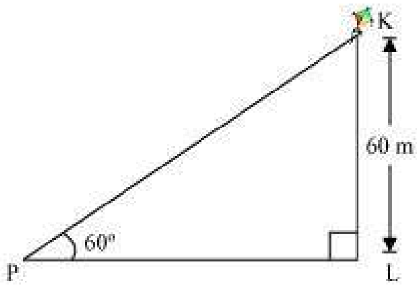
$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m.

Question 5:

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Answer:



Let K be the kite and the string is tied to point P on the ground.

In ΔKLP,

$$\frac{KL}{KP} = \sin 60°$$

$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$

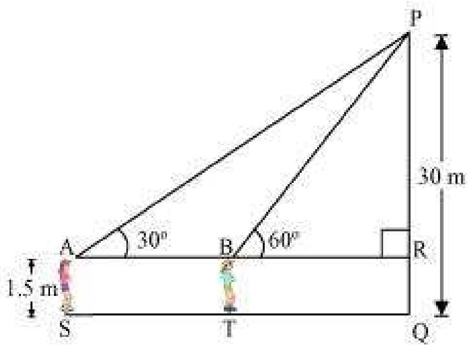
$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is $40\sqrt{3}$ m.

Question 6:

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer:



Let the boy was standing at point S initially. He walked towards the building and reached at point T.

It can be observed that

PR = PQ − RQ

= (30 − 1.5) m = 28.5 m = $\frac{57}{2}$ m

In ΔPAR,

$$\frac{PR}{AR} = \tan 30^\circ$$

$$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$$

$$AR = \left(\frac{57}{2}\sqrt{3}\right) m$$

In ΔPRB,

$$\frac{PR}{BR} = \tan 60°$$

$$\frac{57}{2\ BR} = \sqrt{3}$$

$$BR = \frac{57}{2\sqrt{3}} = \left(\frac{19\sqrt{3}}{2}\right) m$$

ST = AB

$$= AR - BR = \left(\frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2}\right) m$$

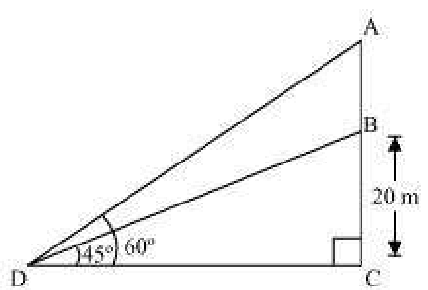
$$= \left(\frac{38\sqrt{3}}{2}\right) m = 19\sqrt{3}\ m$$

Hence, he walked $19\sqrt{3}$ m towards the building.

Question 7:

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Answer:



Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45°$$

$$\frac{20}{CD} = 1$$

$$CD = 20 \text{ m}$$

In ΔACD,

$$\frac{AC}{CD} = \tan 60°$$

$$\frac{AB+BC}{CD} = \sqrt{3}$$

$$\frac{AB+20}{20} = \sqrt{3}$$

$$AB = \left(20\sqrt{3} - 20\right)\text{m}$$

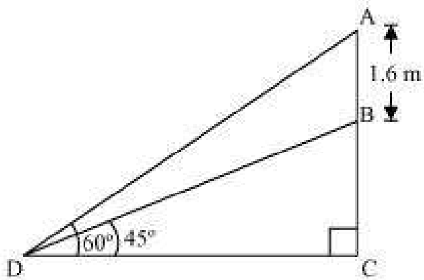
$$= 20\left(\sqrt{3} - 1\right)\text{m}$$

Therefore, the height of the transmission tower is $20\left(\sqrt{3}-1\right)$ m.

Question 8:

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45 °. Find the height of the pedestal.

Answer:



Let AB be the statue, BC be the pedestal, and D be the point on the ground from

In ΔBCD,

$$\frac{BC}{CD} = \tan 45^\circ$$
$$\frac{BC}{CD} = 1$$
$$BC = CD$$

In ΔACD,

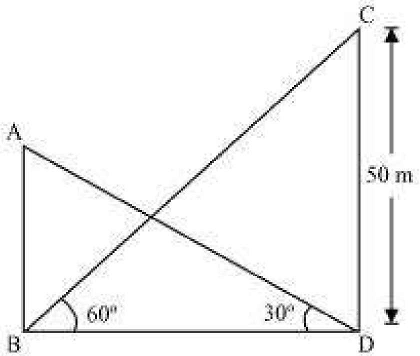
$$\frac{AB+BC}{CD} = \tan 60^\circ$$
$$\frac{AB+BC}{BC} = \sqrt{3}$$
$$1.6 + BC = BC\sqrt{3}$$
$$BC\left(\sqrt{3}-1\right) = 1.6$$
$$BC = \frac{(1.6)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$
$$= \frac{1.6\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2-(1)^2}$$
$$= \frac{1.6\left(\sqrt{3}+1\right)}{2} = 0.8\left(\sqrt{3}+1\right)$$

Therefore, the height of the pedestal is 0.8$\left(\sqrt{3}+1\right)$ m.

Question 9:

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Answer:



Let AB be the building and CD be the tower.

In ΔCDB,

$$\frac{CD}{BD} = \tan 60°$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

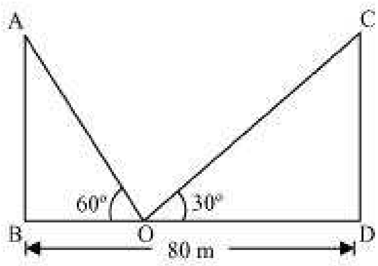
$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

Therefore, the height of the building is $16\frac{2}{3}$ m.

Question 10:

Two poles of equal heights are standing opposite each other an either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30º, respectively. Find the height of poles and the distance of the point from the poles.

Answer:



Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In ΔABO,

$$\frac{AB}{BO} = \tan 60^\circ$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}}$$

In ΔCDO,

$$\frac{CD}{DO} = \tan 30^\circ$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights,

CD = AB

$$CD\left[\sqrt{3}+\frac{1}{\sqrt{3}}\right]=80$$

$$CD\left(\frac{3+1}{\sqrt{3}}\right)=80$$

$$CD=20\sqrt{3}\text{ m}$$

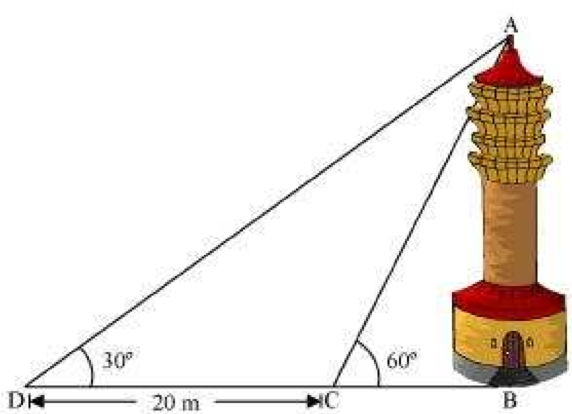
$$BO=\frac{AB}{\sqrt{3}}=\frac{CD}{\sqrt{3}}=\left(\frac{20\sqrt{3}}{\sqrt{3}}\right)\text{m}=20\text{ m}$$

DO = BD − BO = (80 − 20) m = 60 m

Therefore, the height of poles is $20\sqrt{3}$ m and the point is 20 m and 60 m far from these poles.

Question 11:

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



Answer:

In ΔABC,

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{AB}{BC+CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}}+20} = \frac{1}{\sqrt{3}}$$

$$\frac{AB\sqrt{3}}{AB+20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

$$AB = 10\sqrt{3} \text{ m}$$

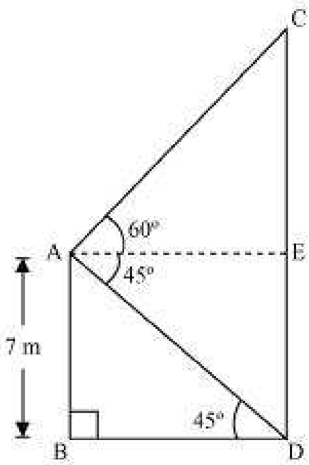
$$BC = \frac{AB}{\sqrt{3}} = \left(\frac{10\sqrt{3}}{\sqrt{3}}\right)\text{m} = 10 \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

Question 12:

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Answer:



Let AB be a building and CD be a cable tower.

In ΔABD,

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{7}{BD} = 1$$

$$BD = 7 \text{ m}$$

In ΔACE,

AC = BD = 7 m

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{CE}{7} = \sqrt{3}$$

$$CE = 7\sqrt{3} \text{ m}$$

$$CD = CE + ED = \left(7\sqrt{3} + 7\right)\text{m}$$

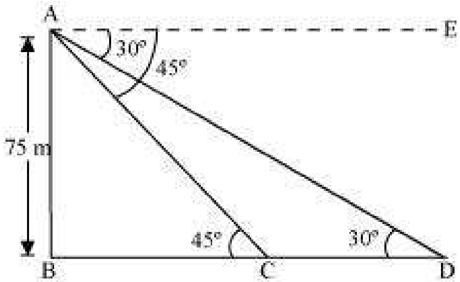
$$= 7\left(\sqrt{3} + 1\right)\text{m}$$

Therefore, the height of the cable tower is $7\left(\sqrt{3}+1\right)$ m.

Question 13:

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer:



Let AB be the lighthouse and the two ships be at point C and D respectively.

In ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{75}{BC} = 1$$

$$BC = 75 \text{ m}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{75}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{75}{75 + CD} = \frac{1}{\sqrt{3}}$$

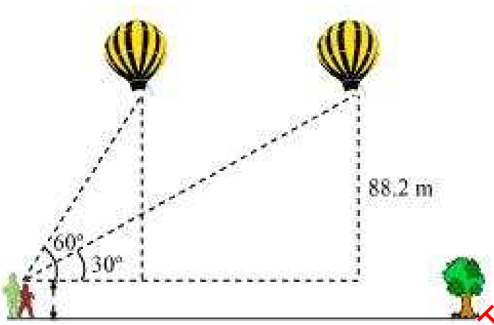
$$75\sqrt{3} = 75 + CD$$

$$75\left(\sqrt{3} - 1\right)\text{m} = CD$$

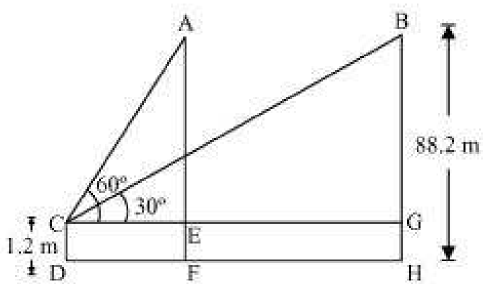
Therefore, the distance between the two ships is $75\left(\sqrt{3} - 1\right)$ m.

Question 14:

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



Answer:



Let the initial position A of balloon change to B after some time and CD be the girl.

In ΔACE,

$$\frac{AE}{CE} = \tan 60^\circ$$

$$\frac{AF - EF}{CE} = \tan 60^\circ$$

$$\frac{88.2 - 1.2}{CE} = \sqrt{3}$$

$$\frac{87}{CE} = \sqrt{3}$$

$$CE = \frac{87}{\sqrt{3}} = 29\sqrt{3} \text{ m}$$

In ΔBCG,

$$\frac{BG}{CG} = \tan 30^\circ$$

$$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$$

$$87\sqrt{3} \text{ m} = CG$$

Distance travelled by balloon = EG = CG − CE

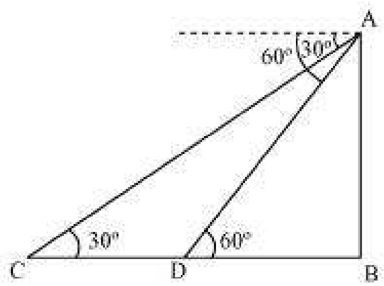
$$= \left(87\sqrt{3} - 29\sqrt{3}\right)\text{m}$$

$$= 58\sqrt{3} \text{ m}$$

Question 15:

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer:



Let AB be the tower.

Initial position of the car is C, which changes to D after six seconds.

In ΔADB,

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In ΔABC,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2AB}{\sqrt{3}}$$

Time taken by the car to travel distance DC $\left(\text{i.e., } \frac{2AB}{\sqrt{3}}\right)$ = 6 seconds

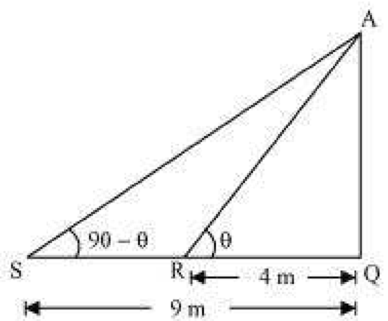
Time taken by the car to travel distance DB $\left(\text{i.e., } \frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}}$

$= \frac{6}{2} = 3$ seconds

Question 16:

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer:



Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.

The angles are complementary. Therefore, if one angle is θ, the other will be 90 − θ.

In ΔAQR,

$$\frac{AQ}{QR} = \tan\theta$$

$$\frac{AQ}{4} = \tan\theta \qquad \ldots(i)$$

In ΔAQS,

$$\frac{AQ}{SQ} = \tan(90 - \theta)$$

$$\frac{AQ}{9} = \cot\theta \qquad ...(ii)$$

On multiplying equations (*i*) and (*ii*), we obtain

$$\left(\frac{AQ}{4}\right)\left(\frac{AQ}{9}\right) = (\tan\theta).(\cot\theta)$$

$$\frac{AQ^2}{36} = 1$$

$$AQ^2 = 36$$

$$AQ = \sqrt{36} = \pm 6$$

However, height cannot be negative.

Therefore, the height of the tower is 6 m.

Exercise 10.2

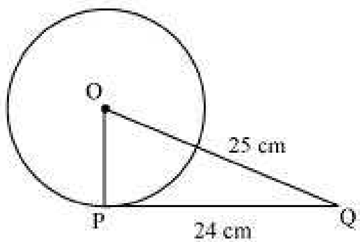
Question 1:

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm

(C) 15 cm (D) 24.5 cm

Answer:



Let O be the centre of the circle.

Given that,

OQ = 25cm and PQ = 24 cm

As the radius is perpendicular to the tangent at the point of contact,

Therefore, OP $\perp$ PQ

Applying Pythagoras theorem in ΔOPQ, we obtain

$OP^2 + PQ^2 = OQ^2$

$OP^2 + 24^2 = 25^2$

$OP^2 = 625 - 576$

$OP^2 = 49$

OP = 7

Therefore, the radius of the circle is 7 cm.

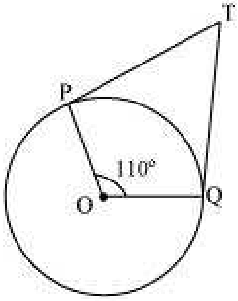
Hence, alternative (A) is correct

Question 2:

In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to

(A) 60° (B) 70°

(C) 80° (D) 90°



Answer:

It is given that TP and TQ are tangents.

Therefore, radius drawn to these tangents will be perpendicular to the tangents.

Thus, OP $\perp$ TP and OQ $\perp$ TQ

$\angle$OPT = 90º

$\angle$OQT = 90º

In quadrilateral POQT,

Sum of all interior angles = 360°

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^{\circ}$

$\Rightarrow 90^{\circ} + 110^{\circ} + 90^{\circ} + \angle PTQ = 360^{\circ}$

$\Rightarrow \angle PTQ = 70^{\circ}$

Hence, alternative (B) is correct

Question 3:

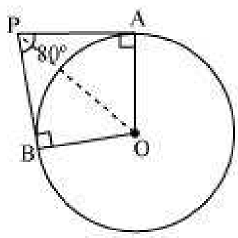
If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80°, then $\angle$POA is equal to

(A) 50° (B) 60°

(C) 70° (D) 80°

Answer:

It is given that PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA $\perp$ PA and OB $\perp$ PB

$\angle$OBP = 90º

$\angle$OAP = 90º

In AOBP,

Sum of all interior angles = 360°

$\angle$OAP + $\angle$APB + $\angle$PBO + $\angle$BOA = 360°

90° + 80° + 90º + $\angle$BOA = 360°

$\angle$BOA = 100°

In ΔOPB and ΔOPA,

AP = BP (Tangents from a point)

OA = OB (Radii of the circle)

OP = OP (Common side)

Therefore, ΔOPB ≅ ΔOPA (SSS congruence criterion)

A ↔ B, P ↔ P, O ↔ O

And thus, $\angle$POB = $\angle$POA

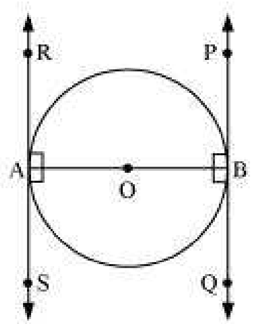
$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Hence, alternative (A) is correct.

Question 4:

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer:



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA $\perp$ RS and OB $\perp$ PQ

$\angle OAR = 90^\circ$

$\angle OAS = 90^\circ$

$\angle OBP = 90^\circ$

$\angle OBQ = 90^\circ$

It can be observed that

$\angle OAR = \angle OBQ$ (Alternate interior angles)

$\angle OAS = \angle OBP$ (Alternate interior angles)

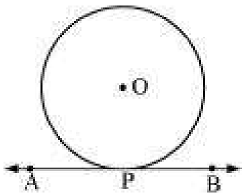
Since alternate interior angles are equal, lines PQ and RS will be parallel.

Question 5:

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

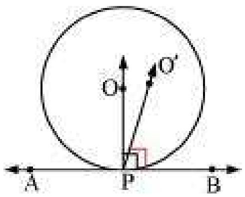
Answer:

Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



We have to prove that the line perpendicular to AB at P passes through centre O. We shall prove this by contradiction method.

Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.



As perpendicular to AB at P passes through O', therefore,

$\angle O'PB = 90°$ … (1)

O is the centre of the circle and P is the point of contact. We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

$\therefore\ \angle OPB = 90°$ … (2)

Comparing equations (1) and (2), we obtain

$\angle O'PB = \angle OPB$ … (3)

From the figure, it can be observed that,

$\angle O'PB < \angle OPB$ … (4)

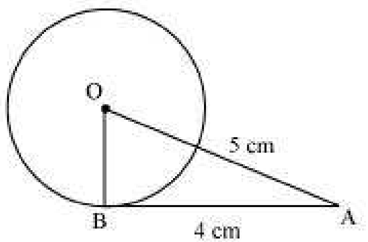
Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O'P coincides with OP.

Therefore, the perpendicular to AB through P passes through centre O.

Question 6:

The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer:



Let us consider a circle centered at point O.

AB is a tangent drawn on this circle from point A.

Given that,

OA = 5cm and AB = 4 cm

In ΔABO,

OB ⊥ AB (Radius ⊥ tangent at the point of contact)

Applying Pythagoras theorem in ΔABO, we obtain

$AB^2 + BO^2 = OA^2$

$4^2 + BO^2 = 5^2$

$16 + BO^2 = 25$

$BO^2 = 9$

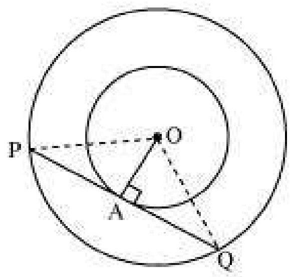
BO = 3

Hence, the radius of the circle is 3 cm.

Question 7:

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer:



Let the two concentric circles be centered at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

OA $\perp$ PQ (As OA is the radius of the circle)

Applying Pythagoras theorem in ΔOAP, we obtain

$OA^2 + AP^2 = OP^2$

$3^2 + AP^2 = 5^2$

$9 + AP^2 = 25$

$AP^2 = 16$

AP = 4

In ΔOPQ,

Since OA $\perp$ PQ,

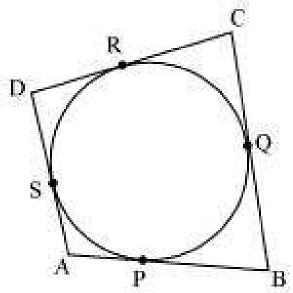
AP = AQ (Perpendicular from the center of the circle bisects the chord)

$\therefore$ PQ = 2AP = 2 × 4 = 8

Therefore, the length of the chord of the larger circle is 8 cm.

Question 8:

A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that AB + CD = AD + BC



Answer:

It can be observed that

DR = DS (Tangents on the circle from point D) … (1)

CR = CQ (Tangents on the circle from point C) … (2)

BP = BQ (Tangents on the circle from point B) … (3)

AP = AS (Tangents on the circle from point A) … (4)

Adding all these equations, we obtain

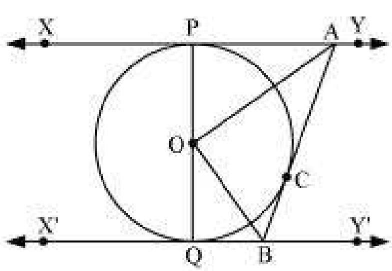
DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

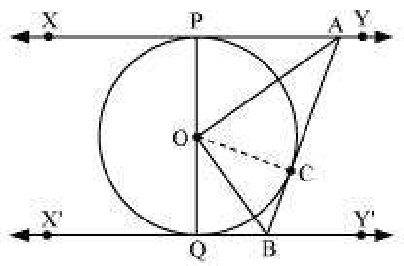
Question 9:

In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.



Answer:

Let us join point O to C



In ΔOPA and ΔOCA,

OP = OC (Radii of the same circle)

AP = AC (Tangents from point A)

AO = AO (Common side)

ΔOPA ≅ ΔOCA (SSS congruence criterion)

Therefore, P ↔ C, A ↔ A, O ↔ O

$\angle$POA = $\angle$COA … (*i*)

Similarly, ΔOQB ≅ ΔOCB

$\angle$QOB = $\angle$COB … (*ii*)

Since POQ is a diameter of the circle, it is a straight line.

Therefore, $\angle$POA + $\angle$COA + $\angle$COB + $\angle$QOB = 180º

From equations (*i*) and (*ii*), it can be observed that

2$\angle$COA + 2 $\angle$COB = 180º

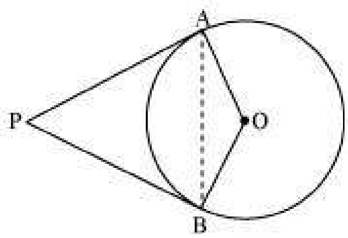
$\angle$COA + $\angle$COB = 90º

$\angle$AOB = 90°

Question 10:

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer:



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends $\angle$AOB at center O of the circle.

It can be observed that

OA (radius) $\perp$ PA (tangent)

Therefore, $\angle OAP = 90°$

Similarly, OB (radius) $\perp$ PB (tangent)

$\angle OBP = 90°$

In quadrilateral OAPB,

Sum of all interior angles = 360º

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$

$90° + \angle APB + 90° + \angle BOA = 360°$

$\angle APB + \angle BOA = 180°$

Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Question 11:

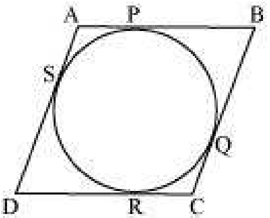
Prove that the parallelogram circumscribing a circle is a rhombus.

Answer:

Since ABCD is a parallelogram,

AB = CD ...(1)

BC = AD ...(2)



It can be observed that

DR = DS (Tangents on the circle from point D)

CR = CQ (Tangents on the circle from point C)

BP = BQ (Tangents on the circle from point B)

AP = AS (Tangents on the circle from point A)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

On putting the values of equations (1) and (2) in this equation, we obtain

2AB = 2BC

AB = BC ...(3)

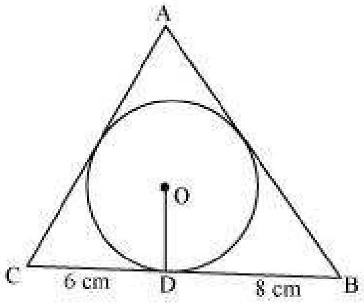
Comparing equations (1), (2), and (3), we obtain

AB = BC = CD = DA

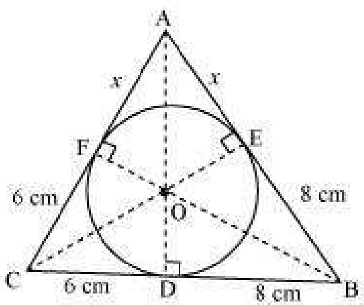
Hence, ABCD is a rhombus.

Question 12:

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see given figure). Find the sides AB and AC.



Answer:



Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be *x.*

In ΔABC,

CF = CD = 6cm (Tangents on the circle from point C)

BE = BD = 8cm (Tangents on the circle from point B)

AE = AF = x (Tangents on the circle from point A)

AB = AE + EB = x + 8

BC = BD + DC = 8 + 6 = 14

CA = CF + FA = 6 + x

2s = AB + BC + CA

= x + 8 + 14 + 6 + x

= 28 + 2x

s = 14 + x

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}}$$
$$= \sqrt{(14+x)(x)(8)(6)}$$
$$= 4\sqrt{3(14x+x^2)}$$

Area of ΔOBC = $\frac{1}{2}\times OD\times BC = \frac{1}{2}\times 4\times 14 = 28$

Area of ΔOCA = $\frac{1}{2}\times OF\times AC = \frac{1}{2}\times 4\times(6+x) = 12+2x$

Area of ΔOAB = $\frac{1}{2}\times OE\times AB = \frac{1}{2}\times 4\times(8+x) = 16+2x$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$4\sqrt{3(14x+x^2)} = 28+12+2x+16+2x$$
$$\Rightarrow 4\sqrt{3(14x+x^2)} = 56+4x$$
$$\Rightarrow \sqrt{3(14x+x^2)} = 14+x$$
$$\Rightarrow 3(14x+x^2) = (14+x)^2$$
$$\Rightarrow 42x+3x^2 = 196+x^2+28x$$
$$\Rightarrow 2x^2+14x-196 = 0$$
$$\Rightarrow x^2+7x-98=0$$
$$\Rightarrow x^2+14x-7x-98=0$$
$$\Rightarrow x(x+14)-7(x+14)=0$$
$$\Rightarrow (x+14)(x-7)=0$$

Either $x+14 = 0$ or $x - 7 = 0$

Therefore, $x = -14$ and 7

However, $x = -14$ is not possible as the length of the sides will be negative.

Therefore, $x = 7$

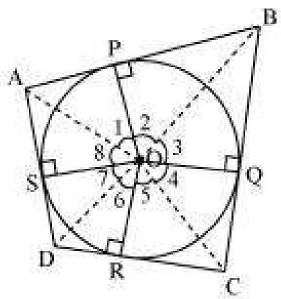
Hence, AB = x + 8 = 7 + 8 = 15 cm

CA = 6 + x = 6 + 7 = 13 cm

Question 13:

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer:



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS (SSS congruence criterion)

Therefore, A ↔ A, P ↔ S, O ↔ O

And thus, ∠POA = ∠AOS

∠1 = ∠8

Similarly,

∠2 = ∠3

∠4 = ∠5

∠6 = ∠7

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^{\circ}$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^{\circ}$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^{\circ}$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^{\circ}$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^{\circ}$

$\angle AOB + \angle COD = 180^{\circ}$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^{\circ}$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle

Exercise 11.1

Question 1:

Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure the two parts. Give the justification of the construction.

Answer:

A line segment of length 7.6 cm can be divided in the ratio of 5:8 as follows.

Step 1 Draw line segment AB of 7.6 cm and draw a ray AX making an acute angle with line segment AB.

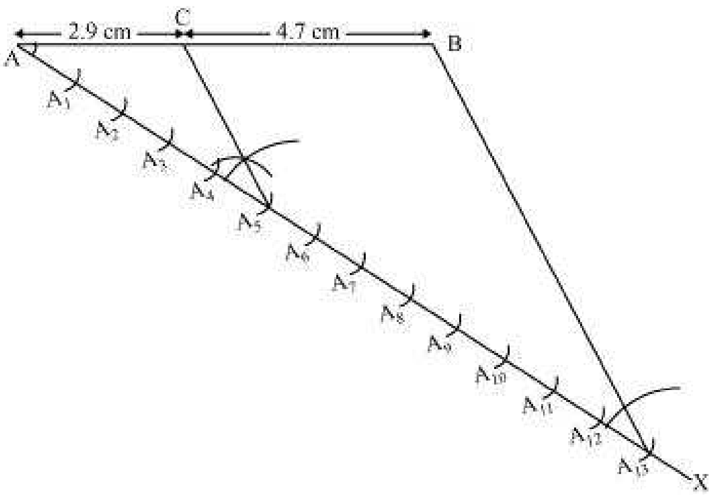
Step 2 Locate 13 (= 5 + 8) points, A_1, A_2, A_3, A_4 …….. A_{13}, on AX such that $AA_1 = A_1A_2 = A_2A_3$ and so on.

Step 3 Join BA_{13}.

Step 4 Through the point A_5, draw a line parallel to BA_{13} (by making an angle equal to $\angle AA_{13}B$) at A_5 intersecting AB at point C.

C is the point dividing line segment AB of 7.6 cm in the required ratio of 5:8.

The lengths of AC and CB can be measured. It comes out to 2.9 cm and 4.7 cm respectively.



Justification

The construction can be justified by proving that

$$\frac{AC}{CB} = \frac{5}{8}$$

By construction, we have $A_5C \parallel A_{13}B$. By applying Basic proportionality theorem for the triangle $AA_{13}B$, we obtain

$$\frac{AC}{CB} = \frac{AA_5}{A_5A_{13}} \quad \dots (1)$$

From the figure, it can be observed that AA_5 and A_5A_{13} contain 5 and 8 equal divisions of line segments respectively.

$$\therefore \frac{AA_5}{A_5A_{13}} = \frac{5}{8} \quad \dots (2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AC}{CB} = \frac{5}{8}$$

This justifies the construction.

Question 2:

Construct a triangle of sides 4 cm, 5cm and 6cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Give the justification of the construction.

Answer:

Step 1

Draw a line segment AB = 4 cm. Taking point A as centre, draw an arc of 5 cm radius. Similarly, taking point B as its centre, draw an arc of 6 cm radius. These arcs will intersect each other at point C. Now, AC = 5 cm and BC = 6 cm and ΔABC is the required triangle.

Step 2

Draw a ray AX making an acute angle with line AB on the opposite side of vertex C.

Step 3

Locate 3 points A_1, A_2, A_3 (as 3 is greater between 2 and 3) on line AX such that $AA_1 = A_1A_2 = A_2A_3$.

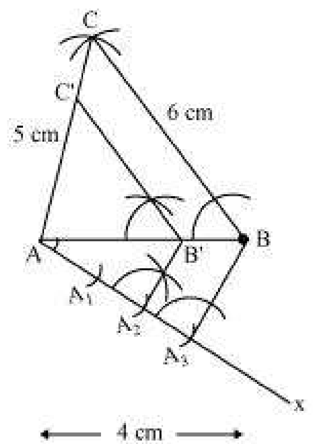
Step 4

Join BA_3 and draw a line through A_2 parallel to BA_3 to intersect AB at point B'.

Step 5

Draw a line through B' parallel to the line BC to intersect AC at C'.

ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{2}{3}AB,\ B'C' = \frac{2}{3}BC,\ AC' = \frac{2}{3}AC$$

By construction, we have B'C' || BC

$\therefore \angle AB'C' = \angle ABC$ (Corresponding angles)

In ΔAB'C' and ΔABC,

$\angle AB'C' = \angle ABC$ (Proved above)

$\angle B'AC' = \angle BAC$ (Common)

$\therefore \Delta AB'C' \sim \Delta ABC$ (AA similarity criterion)

$$\Rightarrow \frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} \quad \dots (1)$$

In $\Delta AA_2B'$ and ΔAA_3B,

$\angle A_2AB' = \angle A_3AB$ (Common)

$\angle AA_2B' = \angle AA_3B$ (Corresponding angles)

$\therefore \Delta AA_2B' \sim \Delta AA_3B$ (AA similarity criterion)

$$\Rightarrow \frac{AB'}{AB} = \frac{AA_2}{AA_3}$$

$$\Rightarrow \frac{AB'}{AB} = \frac{2}{3} \quad \dots (2)$$

From equations (1) and (2), we obtain

$$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{2}{3}$$

$$\Rightarrow AB' = \frac{2}{3}AB,\ B'C' = \frac{2}{3}BC,\ AC' = \frac{2}{3}AC$$

This justifies the construction.

Question 3:

Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Give the justification of the construction.

Answer:

Step 1

Draw a line segment AB of 5 cm. Taking A and B as centre, draw arcs of 6 cm and 5 cm radius respectively. Let these arcs intersect each other at point C. ΔABC is the required triangle having length of sides as 5 cm, 6 cm, and 7 cm respectively.

Step 2

Draw a ray AX making acute angle with line AB on the opposite side of vertex C.

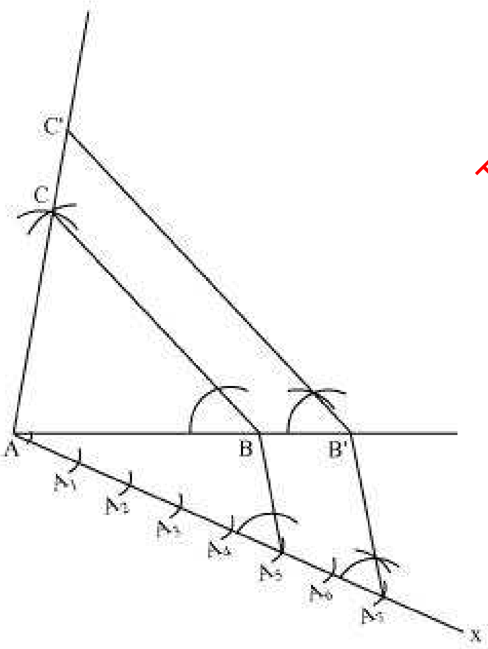
Locate 7 points, A_1, A_2, A_3, A_4 A_5, A_6, A_7 (as 7 is greater between 5and 7), on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

Step 4

Join BA_5 and draw a line through A_7 parallel to BA_5 to intersect extended line segment AB at point B'.

Step 5

Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{7}{5}AB,\ B'C' = \frac{7}{5}BC,\ AC' = \frac{7}{5}AC$$

In ΔABC and ΔAB'C',

$\angle ABC = \angle AB'C'$ (Corresponding angles)

$\angle BAC = \angle B'AC'$ (Common)

$\therefore$ ΔABC ~ ΔAB'C' (AA similarity criterion)

$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$ ... (1)

In ΔAA_5B and $\Delta AA_7B'$,

$\angle A_5AB = \angle A_7AB'$ (Common)

$\angle AA_5B = \angle AA_7B'$ (Corresponding angles)

$\therefore \Delta AA_5B \sim \Delta AA_7B'$ (AA similarity criterion)

$\Rightarrow \frac{AB}{AB'} = \frac{AA_5}{AA_7}$

$\Rightarrow \frac{AB}{AB'} = \frac{5}{7}$ ...(2)

On comparing equations (1) and (2), we obtain

$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{5}{7}$

$\Rightarrow AB' = \frac{7}{5}AB, B'C' = \frac{7}{5}BC, AC' = \frac{7}{5}AC$

This justifies the construction.

Question 4:

Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose side are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Give the justification of the construction.

Answer:

Let us assume that ΔABC is an isosceles triangle having CA and CB of equal lengths, base AB of 8 cm, and AD is the altitude of 4 cm.

A ΔAB'C' whose sides are $\frac{3}{2}$ times of ΔABC can be drawn as follows.

Step 1

Draw a line segment AB of 8 cm. Draw arcs of same radius on both sides of the line segment while taking point A and B as its centre. Let these arcs intersect each other at O and O'. Join OO'. Let OO' intersect AB at D.

Step 2

Taking D as centre, draw an arc of 4 cm radius which cuts the extended line segment OO' at point C. An isosceles ΔABC is formed, having CD (altitude) as 4 cm and AB (base) as 8 cm.

Step 3

Draw a ray AX making an acute angle with line segment AB on the opposite side of vertex C.

Step 4

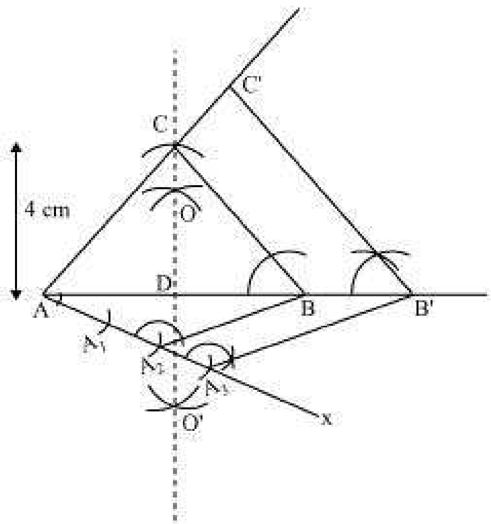
Locate 3 points (as 3 is greater between 3 and 2) A_1, A_2, and A_3 on AX such that $AA_1 = A_1A_2 = A_2A_3$.

Step 5

Join BA_2 and draw a line through A_3 parallel to BA_2 to intersect extended line segment AB at point B'.

Step 6

Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{3}{2}AB,\ B'C' = \frac{3}{2}BC,\ AC' = \frac{3}{2}AC$$

In ΔABC and ΔAB'C',

$\angle$ABC = $\angle$AB'C' (Corresponding angles)

$\angle$BAC = $\angle$B'AC' (Common)

∴ ΔABC ~ ΔAB'C' (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} \quad \ldots (1)$$

In ΔAA_2B and $\Delta AA_3B'$,

$\angle A_2AB = \angle A_3AB'$ (Common)

$\angle AA_2B = \angle AA_3B'$ (Corresponding angles)

∴ $\Delta AA_2B \sim \Delta AA_3B'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_2}{AA_3}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{2}{3} \quad \ldots(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{AB'}=\frac{BC}{B'C'}=\frac{AC}{AC'}=\frac{2}{3}$$

$$\Rightarrow AB'=\frac{3}{2}AB,\ B'C'=\frac{3}{2}BC,\ AC'=\frac{3}{2}AC$$

This justifies the construction.

Question 5:

Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle$ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC. Give the justification of the construction.

Answer:

A ΔA'BC' whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC can be drawn as follows.

Step 1

Draw a ΔABC with side BC = 6 cm, AB = 5 cm and $\angle$ABC = 60°.

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3

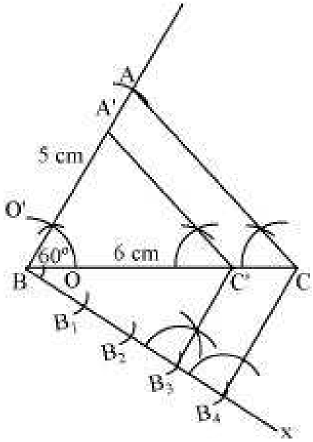
Locate 4 points (as 4 is greater in 3 and 4), B_1, B_2, B_3, B_4, on line segment BX.

Step 4

Join B_4C and draw a line through B_3, parallel to B_4C intersecting BC at C'.

Step 5

Draw a line through C' parallel to AC intersecting AB at A'. ΔA'BC' is the required triangle.



Justification

The construction can be justified by proving

$$A'B = \frac{3}{4}AB,\ BC' = \frac{3}{4}BC,\ A'C' = \frac{3}{4}AC$$

In ΔA'BC' and ΔABC,

∠A'C'B = ∠ACB (Corresponding angles)

∠A'BC' = ∠ABC (Common)

∴ ΔA'BC' ~ ΔABC (AA similarity criterion)

$$\Rightarrow \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} \quad \dots (1)$$

In $\Delta BB_3C'$ and ΔBB_4C,

$\angle B_3BC' = \angle B_4BC$ (Common)

$\angle BB_3C' = \angle BB_4C$ (Corresponding angles)

∴ $\Delta BB_3C' \sim \Delta BB_4C$ (AA similarity criterion)

$$\Rightarrow \frac{BC'}{BC} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC'}{BC} = \frac{3}{4} \quad \dots(2)$$

From equations (1) and (2), we obtain

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{3}{4}$$

$$\Rightarrow A'B = \frac{3}{4}AB,\ BC' = \frac{3}{4}BC,\ A'C' = \frac{3}{4}AC$$

This justifies the construction.

Question 6:

Draw a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding side of ΔABC. Give the justification of the construction.

Answer:

$\angle B = 45°$, $\angle A = 105°$

Sum of all interior angles in a triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

$105° + 45° + \angle C = 180°$

$\angle C = 180° - 150°$

$\angle C = 30°$

The required triangle can be drawn as follows.

Step 1

Draw a ΔABC with side BC = 7 cm, $\angle B = 45°$, $\angle C = 30°$.

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3

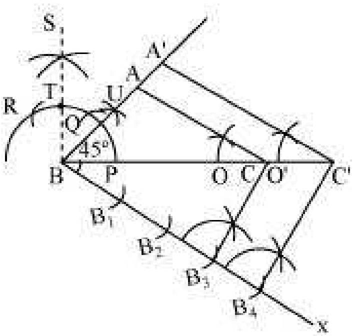
Locate 4 points (as 4 is greater in 4 and 3), B_1, B_2, B_3, B_4, on BX.

Step 4

Join B_3C. Draw a line through B_4 parallel to B_3C intersecting extended BC at C'.

Step 5

Through C', draw a line parallel to AC intersecting extended line segment at C'. ΔA'BC' is the required triangle.



Justification

The construction can be justified by proving that

$$A'B = \frac{4}{3}AB,\ BC' = \frac{4}{3}BC,\ A'C' = \frac{4}{3}AC$$

In ΔABC and ΔA'BC',

$\angle$ABC = $\angle$A'BC' (Common)

$\angle$ACB = $\angle$A'C'B (Corresponding angles)

∴ ΔABC ~ ΔA'BC' (AA similarity criterion)

$$\Rightarrow \frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} \quad \ldots (1)$$

In ΔBB_3C and $\Delta BB_4C'$,

$\angle B_3BC = \angle B_4BC'$ (Common)

$\angle BB_3C = \angle BB_4C'$ (Corresponding angles)

∴ $\Delta BB_3C \sim \Delta BB_4C'$ (AA similarity criterion)

$$\Rightarrow \frac{BC}{BC'} = \frac{BB_3}{BB_4}$$

$$\Rightarrow \frac{BC}{BC'} = \frac{3}{4} \qquad \ldots(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{A'B}=\frac{BC}{BC'}=\frac{AC}{A'C'}=\frac{3}{4}$$

$\Rightarrow$ $A'B=\frac{4}{3}AB,\ BC'=\frac{4}{3}BC,\ A'C'=\frac{4}{3}AC$

This justifies the construction.

Question 7:

Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. the construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle. Give the justification of the construction.

Answer:

It is given that sides other than hypotenuse are of lengths 4 cm and 3 cm. Clearly, these will be perpendicular to each other.

The required triangle can be drawn as follows.

Step 1

Draw a line segment AB = 4 cm. Draw a ray SA making 90° with it.

Step 2

Draw an arc of 3 cm radius while taking A as its centre to intersect SA at C. Join BC. ΔABC is the required triangle.

Step 3

Draw a ray AX making an acute angle with AB, opposite to vertex C.

Step 4

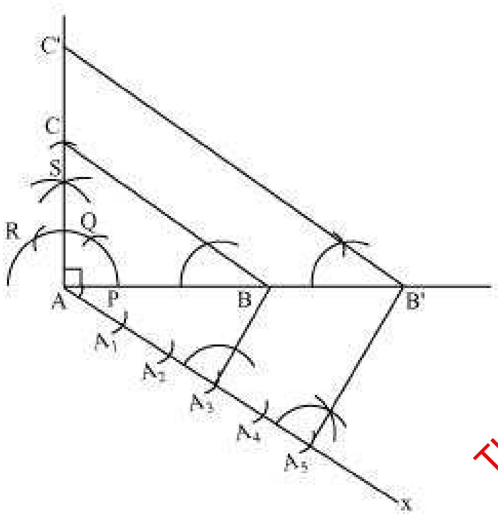
Locate 5 points (as 5 is greater in 5 and 3), A_1, A_2, A_3, A_4, A_5, on line segment AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

Step 5

Join A_3B. Draw a line through A_5 parallel to A_3B intersecting extended line segment AB at B'.

Step 6

Through B', draw a line parallel to BC intersecting extended line segment AC at C'. ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{5}{3}AB,\ B'C' = \frac{5}{3}BC,\ AC' = \frac{5}{3}AC$$

In ΔABC and ΔAB'C',

$\angle ABC = \angle AB'C'$ (Corresponding angles)

$\angle BAC = \angle B'AC'$ (Common)

$\therefore$ ΔABC ~ ΔAB'C' (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} \quad \dots (1)$$

In ΔAA_3B and $\Delta AA_5B'$,

$\angle A_3AB = \angle A_5AB'$ (Common)

$\angle AA_3B = \angle AA_5B'$ (Corresponding angles)

$\therefore \Delta AA_3B \sim \Delta AA_5B'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_3}{AA_5}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{3}{5} \quad \ldots(2)$$

On comparing equations (1) and (2), we obtain

$$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{3}{5}$$

$$\Rightarrow AB' = \frac{5}{3}AB,\ B'C' = \frac{5}{3}BC,\ AC' = \frac{5}{3}AC$$

This justifies the construction.

Exercise 11.2

Question 1:

Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Give the justification of the construction.

Answer:

A pair of tangents to the given circle can be constructed as follows.

Step 1

Taking any point O of the given plane as centre, draw a circle of 6 cm radius. Locate a point P, 10 cm away from O. Join OP.

Step 2

Bisect OP. Let M be the mid-point of PO.

Step 3

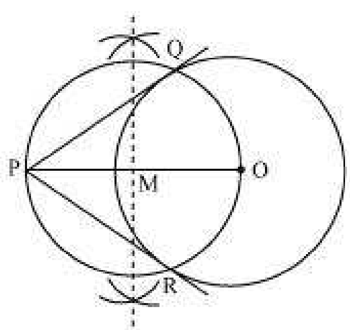
Taking M as centre and MO as radius, draw a circle.

Step 4

Let this circle intersect the previous circle at point Q and R.

Step 5

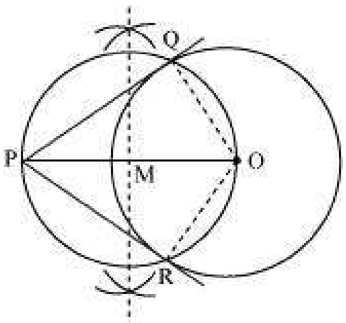
Join PQ and PR. PQ and PR are the required tangents.



The lengths of tangents PQ and PR are 8 cm each.

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 6 cm). For this, join OQ and OR.



$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle PQO = 90°$

$\Rightarrow OQ \perp PQ$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle

Question 2:

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation. Give the justification of the construction.

Answer:

Tangents on the given circle can be drawn as follows.

Step 1

Draw a circle of 4 cm radius with centre as O on the given plane.

Step 2

Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP.

Step 3

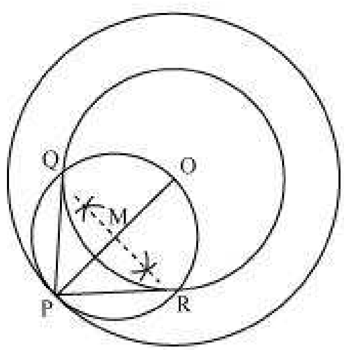
Bisect OP. Let M be the mid-point of PO.

Step 4

Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at the points Q and R.

Step 5

Join PQ and PR. PQ and PR are the required tangents.



It can be observed that PQ and PR are of length 4.47 cm each.

In ΔPQO,

Since PQ is a tangent,

∠PQO = 90°

PO = 6 cm

QO = 4 cm

Applying Pythagoras theorem in ΔPQO, we obtain

$PQ^2 + QO^2 = PQ^2$

$PQ^2 + (4)^2 = (6)^2$

$PQ^2 + 16 = 36$

$PQ^2 = 36 - 16$

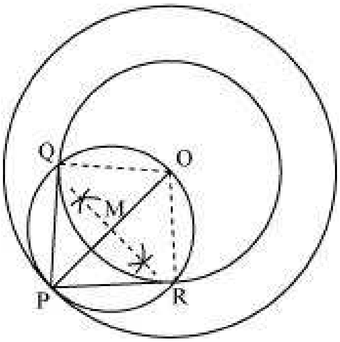
$PQ^2 = 20$

PQ $= 2\sqrt{5}$

PQ = 4.47 cm

Justification

The construction can be justified by proving that PQ and PR are the tangents to the circle (whose centre is O and radius is 4 cm). For this, let us join OQ and OR.



$\angle PQO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle PQO = 90°$

$\Rightarrow OQ \perp PQ$

Since OQ is the radius of the circle, PQ has to be a tangent of the circle. Similarly, PR is a tangent of the circle

Question 3:

Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q. Give the justification of the construction.

Answer:

The tangent can be constructed on the given circle as follows.

Step 1

Taking any point O on the given plane as centre, draw a circle of 3 cm radius.

Step 2

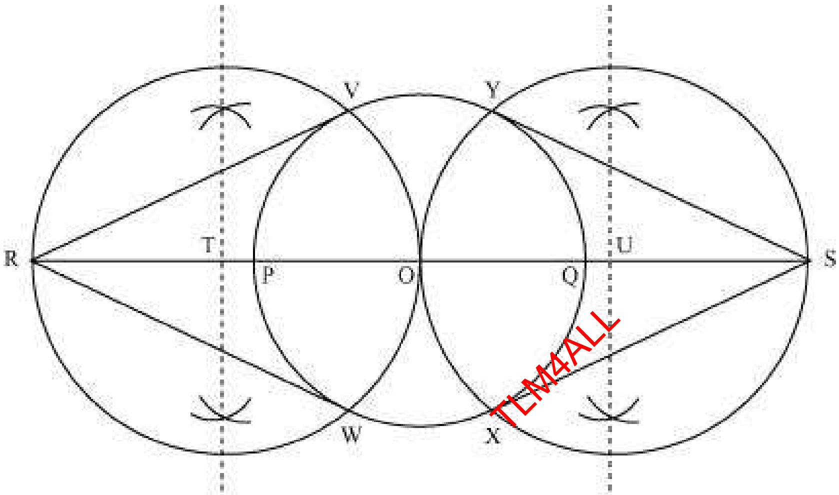
Take one of its diameters, PQ, and extend it on both sides. Locate two points on this diameter such that OR = OS = 7 cm

Step 3

Bisect OR and OS. Let T and U be the mid-points of OR and OS respectively.

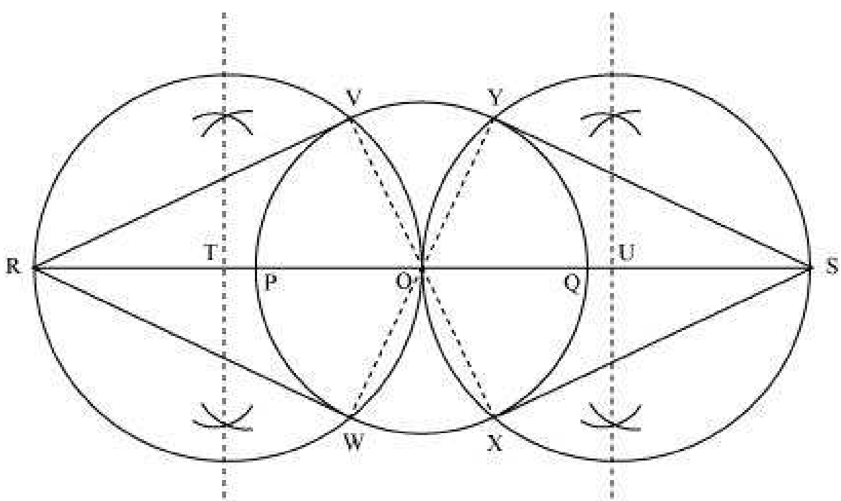
Step 4

Taking T and U as its centre and with TO and UO as radius, draw two circles. These two circles will intersect the circle at point V, W, X, Y respectively. Join RV, RW, SX, and SY. These are the required tangents.



Justification

The construction can be justified by proving that RV, RW, SY, and SX are the tangents to the circle (whose centre is O and radius is 3 cm). For this, join OV, OW, OX, and OY.



$\angle RVO$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.

$\therefore \angle RVO = 90°$

$\Rightarrow OV \perp RV$

Since OV is the radius of the circle, RV has to be a tangent of the circle. Similarly, OW, OX, and OY are the tangents of the circle

Question 4:

Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°. Give the justification of the construction.

Answer:

The tangents can be constructed in the following manner:

Step 1

Draw a circle of radius 5 cm and with centre as O.

Step 2

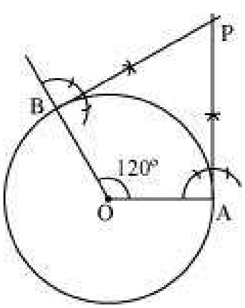
Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

Step 3

Draw a radius OB, making an angle of 120° (180° − 60°) with OA.

Step 4

Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P. PA and PB are the required tangents at an angle of 60°.



Justification

The construction can be justified by proving that $\angle APB = 60°$

By our construction

$\angle OAP = 90°$

$\angle OBP = 90°$

And $\angle AOB = 120°$

We know that the sum of all interior angles of a quadrilateral = 360°

$\angle OAP + \angle AOB + \angle OBP + \angle APB = 360°$

$90° + 120° + 90° + \angle APB = 360°$

$\angle APB = 60°$

This justifies the construction.

Question 5:

Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. Give the justification of the construction.

Answer:

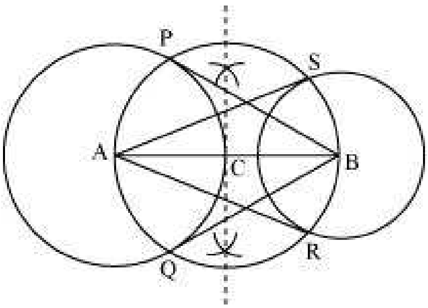
The tangents can be constructed on the given circles as follows.

Step 1

Draw a line segment AB of 8 cm. Taking A and B as centre, draw two circles of 4 cm and 3 cm radius.

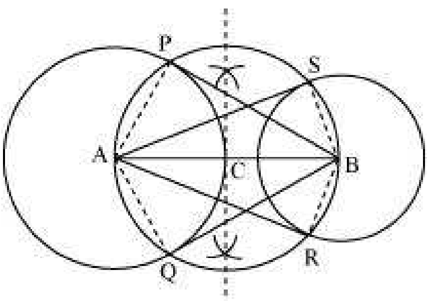
Step 2

Bisect the line AB. Let the mid-point of AB be C. Taking C as centre, draw a circle of AC radius which will intersect the circles at points P, Q, R, and S. Join BP, BQ, AS, and AR. These are the required tangents.



Justification

The construction can be justified by proving that AS and AR are the tangents of the circle (whose centre is B and radius is 3 cm) and BP and BQ are the tangents of the circle (whose centre is A and radius is 4 cm). For this, join AP, AQ, BS, and BR.



$\angle$ASB is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$\therefore \angle ASB = 90°$

$\Rightarrow BS \perp AS$

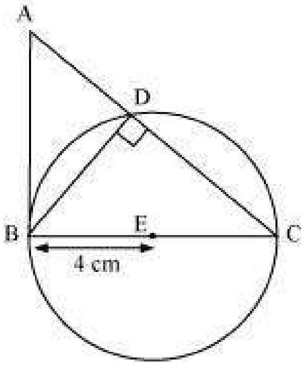
Since BS is the radius of the circle, AS has to be a tangent of the circle. Similarly, AR, BP, and BQ are the tangents.

Question 6:

Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and $\angle$B = 90°. BD is the perpendicular from B on AC. The circle through B, C, and D is drawn. Construct the tangents from A to this circle. Give the justification of the construction.

Answer:

Consider the following situation. If a circle is drawn through B, D, and C, BC will be its diameter as ∠BDC is of measure 90°. The centre E of this circle will be the mid-point of BC.



The required tangents can be constructed on the given circle as follows.

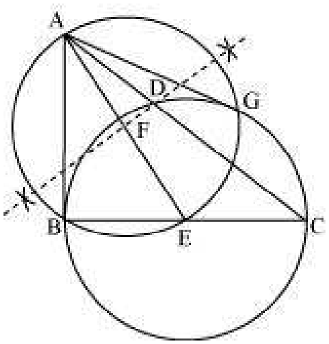
Step 1

Join AE and bisect it. Let F be the mid-point of AE.

Step 2

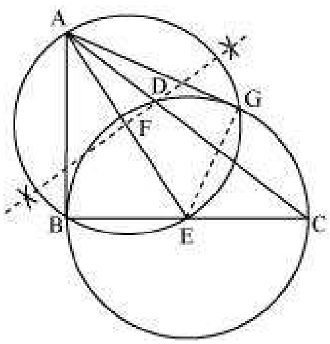
Taking F as centre and FE as its radius, draw a circle which will intersect the circle at point B and G. Join AG.

AB and AG are the required tangents.



Justification

The construction can be justified by proving that AG and AB are the tangents to the circle. For this, join EG.



$\angle AGE$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$\therefore \angle AGE = 90°$

$\Rightarrow EG \perp AG$

Since EG is the radius of the circle, AG has to be a tangent of the circle.

Already, $\angle B = 90°$

$\Rightarrow AB \perp BE$

Since BE is the radius of the circle, AB has to be a tangent of the circle.

Question 7:

Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circles. Give the justification of the construction.

Answer:

The required tangents can be constructed on the given circle as follows.

Step 1

Draw a circle with the help of a bangle.

Step 2

Take a point P outside this circle and take two chords QR and ST.

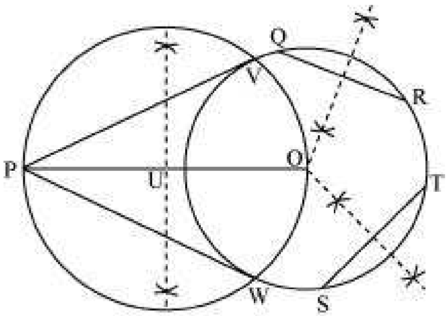
Step 3

Draw perpendicular bisectors of these chords. Let them intersect each other at point O.

Step 4

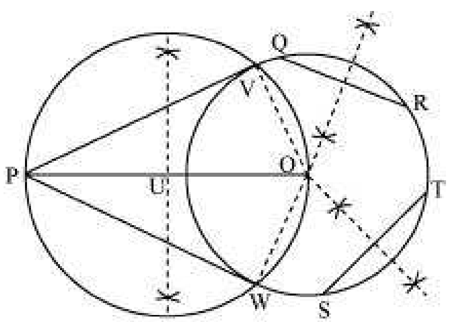
Join PO and bisect it. Let U be the mid-point of PO. Taking U as centre, draw a circle of radius OU, which will intersect the circle at V and W. Join PV and PW.

PV and PW are the required tangents.



Justification

The construction can be justified by proving that PV and PW are the tangents to the circle. For this, first of all, it has to be proved that O is the centre of the circle. Let us join OV and OW.



We know that perpendicular bisector of a chord passes through the centre. Therefore, the perpendicular bisector of chords QR and ST pass through the centre. It is clear that the intersection point of these perpendicular bisectors is the centre of the circle. $\angle PVO$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.

$\therefore \angle PVO = 90°$

$\Rightarrow OV \perp PV$

Since OV is the radius of the circle, PV has to be a tangent of the circle. Similarly, PW is a tangent of the circle.

Exercise 12.1

Question 1:

The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Answer:

Radius (r_1) of 1st circle = 19 cm

Radius (r_2) or 2nd circle = 9 cm

Let the radius of 3rd circle be r.

Circumference of 1st circle = $2\pi r_1 = 2\pi\ (19) = 38\pi$

Circumference of 2nd circle = $2\pi r_2 = 2\pi\ (9) = 18\pi$

Circumference of 3rd circle = $2\pi r$

Given that,

Circumference of 3rd circle = Circumference of 1st circle + Circumference of 2nd circle

$2\pi r = 38\pi + 18\pi = 56\pi$

$$r = \frac{56\pi}{2\pi} = 28$$

Therefore, the radius of the circle which has circumference equal to the sum of the circumference of the given two circles is 28 cm.

Question 2:

The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Answer:

Radius (r_1) of 1st circle = 8 cm

Radius (r_2) of 2nd circle = 6 cm

Let the radius of 3rd circle be r.

Area of 1st circle $= \pi r_1^2 = \pi(8)^2 = 64\pi$

Area of 2nd circle $= \pi r_2^2 = \pi(6)^2 = 36\pi$

Given that,

Area of 3rd circle = Area of 1st circle + Area of 2nd circle

$$\pi r^2 = \pi r_1^2 + \pi r_2^2$$

$$\pi r^2 = 64\pi + 36\pi$$

$$\pi r^2 = 100\pi$$

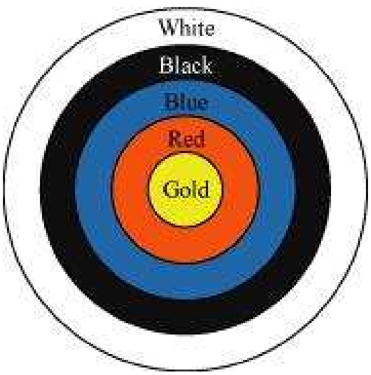
$$r^2 = 100$$

$$r = \pm 10$$

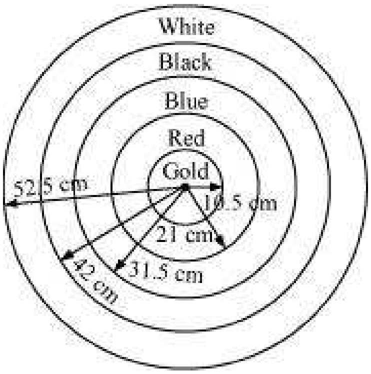
However, the radius cannot be negative. Therefore, the radius of the circle having area equal to the sum of the areas of the two circles is 10 cm.

Question 3:

Given figure depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Radius (r_1) of gold region (i.e., 1st circle) $= \frac{21}{2} = 10.5$ cm

Given that each circle is 10.5 cm wider than the previous circle.

Therefore, radius (r_2) of 2nd circle = 10.5 + 10.5

21 cm

Radius (r_3) of 3rd circle = 21 + 10.5

= 31.5 cm

Radius (r_4) of 4th circle = 31.5 + 10.5

= 42 cm

Radius (r_5) of 5th circle = 42 + 10.5

= 52.5 cm

Area of gold region = Area of 1st circle $= \pi r_1^2 = \pi(10.5)^2 = 346.5 \text{ cm}^2$

Area of red region = Area of 2nd circle − Area of 1st circle

$= \pi r_2^2 - \pi r_1^2$

$= \pi(21)^2 - \pi(10.5)^2$

$= 441\pi - 110.25\pi = 330.75\pi$

$= 1039.5 \text{ cm}^2$

Area of blue region = Area of 3rd circle − Area of 2nd circle

$=\pi r_3^2-\pi r_1^2$

$=\pi(31.5)^2-\pi(21)^2$

$=992.25\pi-441\pi=551.25\pi$

$=1732.5\text{ cm}^2$

Area of black region = Area of 4th circle − Area of 3rd circle

$=\pi r_4^2-\pi r_3^2$

$=\pi(42)^2-\pi(31.5)^2$

$=1764\pi-992.25\pi$

$=771.75\pi=2425.5\text{ cm}^2$

Area of white region = Area of 5th circle − Area of 4th circle

$=\pi r_5^2-\pi r_4^2$

$=\pi(52.5)^2-\pi(42)^2$

$=2756.25\pi-1764\pi$

$=992.25\pi=3118.5\text{ cm}^2$

Therefore, areas of gold, red, blue, black, and white regions are 346.5 cm^2, 1039.5 cm^2, 1732.5 cm^2, 2425.5 cm^2, and 3118.5 cm^2 respectively.

Question 4:

The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is traveling at a speed of 66 km per hour? $\left[\text{Use } \pi=\frac{22}{7}\right]$

Answer:

Diameter of the wheel of the car = 80 cm

Radius (r) of the wheel of the car = 40 cm

Circumference of wheel = $2\pi r$

= $2\pi(40) = 80\pi$ cm

Speed of car = 66 km/hour

$$= \frac{66 \times 100000}{60} \text{ cm/min}$$

$$= 110000 \text{ cm/min}$$

Distance travelled by the car in 10 minutes

= 110000 × 10 = 1100000 cm

Let the number of revolutions of the wheel of the car be *n*.

n × Distance travelled in 1 revolution (i.e., circumference)

= Distance travelled in 10 minutes

$$n \times 80\pi = 1100000$$

$$n = \frac{1100000 \times 7}{80 \times 22}$$

$$= \frac{35000}{8} = 4375$$

Therefore, each wheel of the car will make 4375 revolutions.

Question 5:

Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units (B) π units (C) 4 units (D)7 units

Answer:

Let the radius of the circle be *r*.

Circumference of circle = $2\pi r$

Area of circle = πr^2

Given that, the circumference of the circle and the area of the circle are equal.

This implies $2\pi r = \pi r^2$

$2 = r$

Therefore, the radius of the circle is 2 units.

Hence, the correct answer is A.

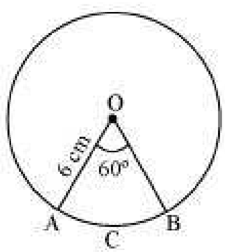
Exercise 12.2

Question 1:

Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Let OACB be a sector of the circle making 60° angle at centre O of the circle.

Area of sector of angle θ = $\frac{\theta}{360°} \times \pi r^2$

Area of sector OACB = $\frac{60°}{360°} \times \frac{22}{7} \times (6)^2$

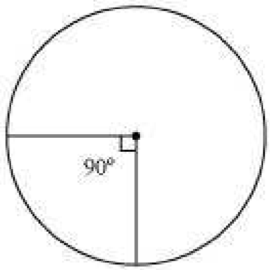
$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2$

Therefore, the area of the sector of the circle making 60° at the centre of the circle is $\frac{132}{7} \text{ cm}^2$

Question 2:

Find the area of a quadrant of a circle whose circumference is 22 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Let the radius of the circle be *r*.

Circumference = 22 cm

$2\pi r = 22$

$$r = \frac{22}{2\pi} = \frac{11}{\pi}$$

Quadrant of circle will subtend 90° angle at the centre of the circle.

Area of such quadrant of the circle $= \frac{90°}{360°} \times \pi \times r^2$

$$= \frac{1}{4\pi} \times \pi \times \left(\frac{11}{}\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8}\ \text{cm}^2$$

Question 3:

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:

We know that in 1 hour (i.e., 60 minutes), the minute hand rotates 360°.

In 5 minutes, minute hand will rotate = $\frac{360^\circ}{60}\times 5 = 30^\circ$

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

Area of sector of angle θ = $\frac{\theta}{360^\circ}\times \pi r^2$

Area of sector of 30° $= \frac{30^\circ}{360^\circ}\times\frac{22}{7}\times 14\times 14$

$= \frac{22}{12}\times 2\times 14$

$= \frac{11\times 14}{3}$

$= \frac{154}{3}\text{ cm}^2$

Therefore, the area swept by the minute hand in 5 minutes is $\frac{154}{3}\text{ cm}^2$.

Question 4:

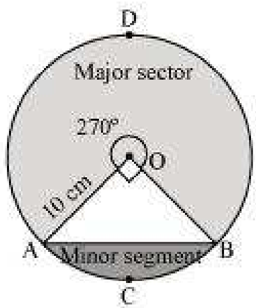
A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector

[Use π = 3.14]

Answer:



Let AB be the chord of the circle subtending 90° angle at centre O of the circle.

Area of major sector OADB = $\left(\frac{360°-90°}{360°}\right)\times\pi r^2=\left(\frac{270°}{360°}\right)\pi r^2$

$=\frac{3}{4}\times 3.14\times 10\times 10$

$=235.5\text{ cm}^2$

Area of minor sector OACB = $\frac{90°}{360°}\times\pi r^2$

$=\frac{1}{4}\times 3.14\times 10\times 10$

$=78.5\text{ cm}^2$

Area of ΔOAB = $\frac{1}{2}\times \text{OA}\times\text{OB}=\frac{1}{2}\times 10\times 10$

= 50 cm^2

Area of minor segment ACB = Area of minor sector OACB −

Area of ΔOAB = 78.5 − 50 = 28.5 cm^2

Question 5:

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) The length of the arc

(ii) Area of the sector formed by the arc

(iii) Area of the segment forced by the corresponding chord

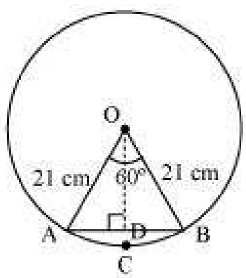
$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of circle = 21 cm

Angle subtended by the given arc = 60°

Length of an arc of a sector of angle θ = $\frac{\theta}{360°} \times 2\pi r$



Length of arc ACB = $\frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$

$= \frac{1}{6} \times 2 \times 22 \times 3$

= 22 cm

Area of sector OACB = $\frac{60°}{360°} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

$= 231 \text{ cm}^2$

In ΔOAB,

∠OAB = ∠OBA (As OA = OB)

∠OAB + ∠AOB + ∠OBA = 180°

2∠OAB + 60° = 180°

∠OAB = 60°

Therefore, ΔOAB is an equilateral triangle.

Area of ΔOAB = $\frac{\sqrt{3}}{4}\times(\text{Side})^2$

$=\frac{\sqrt{3}}{4}\times(21)^2=\frac{441\sqrt{3}}{4}\text{ cm}^2$

Area of segment ACB = Area of sector OACB − Area of ΔOAB

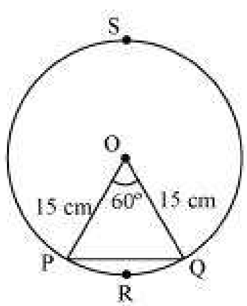
$=\left(231-\frac{441\sqrt{3}}{4}\right)\text{ cm}^2$

Question 6:

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

[Use π = 3.14 and $\sqrt{3}=1.73$]

Answer:



Radius (r) of circle = 15 cm

Area of sector OPRQ = $\frac{60°}{360°}\times\pi r^2$

$=\frac{1}{6}\times3.14\times(15)^2$

$=117.75\text{ cm}^2$

In ΔOPQ,

∠OPQ = ∠OQP (As OP = OQ)

$\angle$OPQ + $\angle$OQP + $\angle$POQ = 180°

2 $\angle$OPQ = 120°

$\angle$OPQ = 60°

ΔOPQ is an equilateral triangle.

Area of ΔOPQ = $\frac{\sqrt{3}}{4}\times(\text{side})^2$

$= \frac{\sqrt{3}}{4}\times(15)^2 = \frac{225\sqrt{3}}{4}\ \text{cm}^2$

$= 56.25\sqrt{3}$

$= 97.3125\ \text{cm}^2$

Area of segment PRQ = Area of sector OPRQ − Area of ΔOPQ

= 117.75 − 97.3125

= 20.4375 cm^2

Area of major segment PSQ = Area of circle − Area of segment PRQ

$= \pi(15)^2 - 20.4375$

$= 3.14\times 225 - 20.4375$

$= 706.5 - 20.4375$

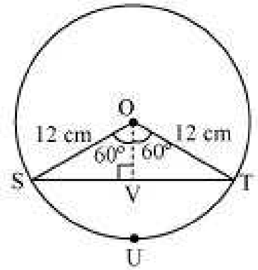
$= 686.0625\ \text{cm}^2$

Question 7:

A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.

[Use π = 3.14 and $\sqrt{3} = 1.73$]

Answer:



Let us draw a perpendicular OV on chord ST. It will bisect the chord ST.

SV = VT

In ΔOVS,

$$\frac{OV}{OS} = \cos 60^\circ$$

$$\frac{OV}{12} = \frac{1}{2}$$

$$OV = 6 \text{ cm}$$

$$\frac{SV}{SO} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{SV}{12} = \frac{\sqrt{3}}{2}$$

$$SV = 6\sqrt{3} \text{ cm}$$

$$ST = 2SV = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$$

Area of ΔOST = $\frac{1}{2} \times ST \times OV$

$$= \frac{1}{2} \times 12\sqrt{3} \times 6$$

$$= 36\sqrt{3} = 36 \times 1.73 = 62.28 \text{ cm}^2$$

Area of sector OSUT = $\frac{120^\circ}{360^\circ} \times \pi (12)^2$

$$= \frac{1}{3} \times 3.14 \times 144 = 150.72 \text{ cm}^2$$

Area of segment SUT = Area of sector OSUT − Area of ΔOST

= 150.72 − 62.28

= 88.44 cm^2

Question 8:

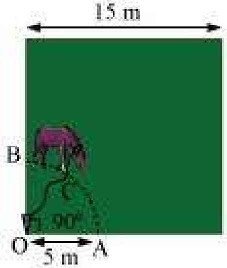
A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see the given figure). Find

(i) The area of that part of the field in which the horse can graze.

(ii) The increase in the grazing area of the rope were 10 m long instead of 5 m.

[Use π = 3.14]

Answer:



From the figure, it can be observed that the horse can graze a sector of 90° in a circle of 5 m radius.

Area that can be grazed by horse = Area of sector OACB

$$= \frac{90^\circ}{360^\circ}\pi r^2$$

$$= \frac{1}{4} \times 3.14 \times (5)^2$$

$$= 19.625 \text{ m}^2$$

Area that can be grazed by the horse when length of rope is 10 m long

$$= \frac{90°}{360°} \times \pi \times (10)^2$$

$$= \frac{1}{4} \times 3.14 \times 100$$

$$= 78.5 \text{ m}^2$$

Increase in grazing area = (78.5 − 19.625) m^2

= 58.875 m^2

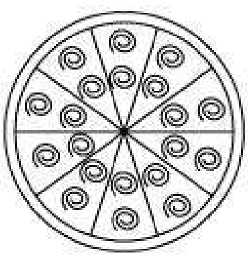
Question 9:

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find.

(i) The total length of the silver wire required.

(ii) The area of each sector of the brooch

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$



Answer:

Total length of wire required will be the length of 5 diameters and the circumference of the brooch.

Radius of circle = $\frac{35}{2}$ mm

Circumference of brooch = $2\pi r$

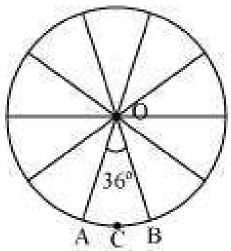
$= 2 \times \frac{22}{7} \times \left(\frac{35}{2}\right)$

= 110 mm

Length of wire required = 110 + 5 × 35

= 110 + 175 = 285 mm

It can be observed from the figure that each of 10 sectors of the circle is subtending 36° at the centre of the circle.



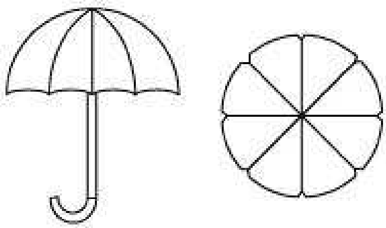
Therefore, area of each sector = $\frac{36°}{360°} \times \pi r^2$

$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$

$= \frac{385}{4} \text{ mm}^2$

Question 10:

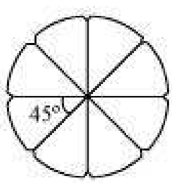
An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

There are 8 ribs in an umbrella. The area between two consecutive ribs is subtending

$\frac{360°}{8} = 45°$ at the centre of the assumed flat circle.



Area between two consecutive ribs of circle = $\frac{45°}{360°} \times \pi r^2$

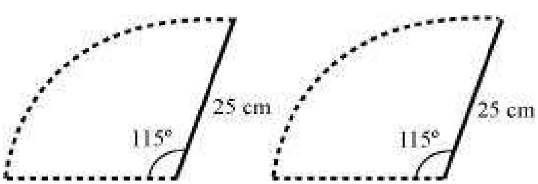
$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$

$= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{ cm}^2$

Question 11:

A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



It can be observed from the figure that each blade of wiper will sweep an area of a sector of 115° in a circle of 25 cm radius.

Area of such sector = $\frac{115°}{360°} \times \pi \times (25)^2$

$= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25$

$= \frac{158125}{252}$ cm^2

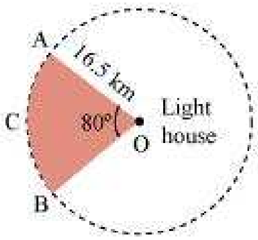
Area swept by 2 blades = $2 \times \frac{158125}{252}$

$= \frac{158125}{126}$ cm^2

Question 12:

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships warned. [Use π = 3.14]

Answer:



It can be observed from the figure that the lighthouse spreads light across a sector of 80° in a circle of 16.5 km radius.

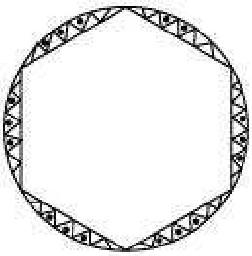
Area of sector OACB = $\frac{80°}{360°} \times \pi r^2$

$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$

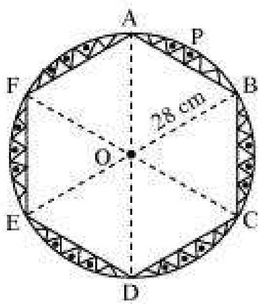
$= 189.97 \text{ km}^2$

Question 13:

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs.0.35 per cm^2. [Use $\sqrt{3} = 1.7$]



Answer:



It can be observed that these designs are segments of the circle.

Consider segment APB. Chord AB is a side of the hexagon. Each chord will substitute $\frac{360°}{6} = 60°$ at the centre of the circle.

In ΔOAB,

$\angle OAB = \angle OBA$ (As OA = OB)

$\angle AOB = 60°$

$\angle OAB + \angle OBA + \angle AOB = 180°$

$2\angle OAB = 180° - 60° = 120°$

$\angle OAB = 60°$

Therefore, ΔOAB is an equilateral triangle.

Area of ΔOAB = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3} = 196 \times 1.7$ = 333.2 cm^2

Area of sector OAPB = $\frac{60°}{360°} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28$

$= \frac{1232}{3}\ cm^2$

Area of segment APB = Area of sector OAPB − Area of ΔOAB

$= \left(\frac{1232}{3} - 333.2\right) cm^2$

$$\text{Therefore, area of designs} = 6 \times \left(\frac{1232}{3} - 333.2\right) cm^2$$

$$= (2464 - 1999.2)\ cm^2$$

$$= 464.8\ cm^2$$

Cost of making 1 cm^2 designs = Rs 0.35

Cost of making 464.76 cm^2 designs = 464.8×0.35 = Rs 162.68

Therefore, the cost of making such designs is Rs 162.68.

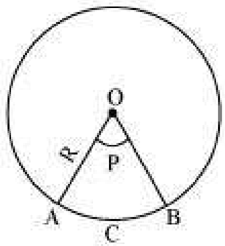
Question 14:

Tick the correct answer in the following:

Area of a sector of angle *p* (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$, (B) $\frac{p}{180} \times \pi R^2$, (C) $\frac{p}{360} \times 2\pi R$, (D) $\frac{p}{720} \times 2\pi R^2$

Answer:



We know that area of sector of angle θ = $\frac{\theta}{360°} \times \pi R^2$

Area of sector of angle P = $\frac{p}{360°}\left(\pi R^2\right)$

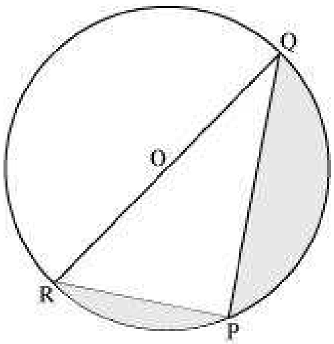
$= \left(\frac{p}{720°}\right)\left(2\pi R^2\right)$

Hence, (D) is the correct answer.

Exercise 12.3

Question 1:

Find the area of the shaded region in the given figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

It can be observed that RQ is the diameter of the circle. Therefore, ∠RPQ will be 90º.

By applying Pythagoras theorem in ΔPQR,

$RP^2 + PQ^2 = RQ^2$

$(7)^2 + (24)^2 = RQ^2$

$RQ = \sqrt{625} = 25$

Radius of circle, $OR = \frac{RQ}{2} = \frac{25}{2}$

Since RQ is the diameter of the circle, it divides the circle in two equal parts.

$$\text{Area of semi-circle RPQOR} = \frac{1}{2}\pi r^2$$
$$= \frac{1}{2}\pi\left(\frac{25}{2}\right)^2$$
$$= \frac{1}{2}\times\frac{22}{7}\times\frac{625}{4}$$
$$= \frac{6875}{28}\text{ cm}^2$$

Area of ΔPQR $= \frac{1}{2}\times PQ\times PR$

$= \frac{1}{2}\times 24\times 7$

$= 84 \text{ cm}^2$

Area of shaded region = Area of semi-circle RPQOR − Area of ΔPQR

$= \frac{6875}{28} - 84$

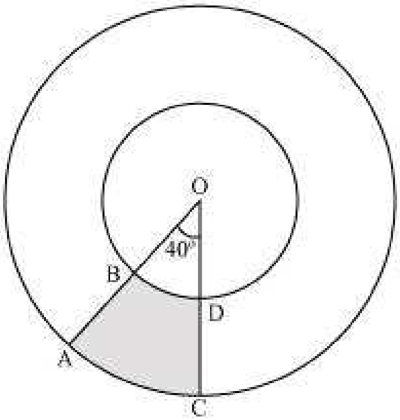
$= \frac{6875-2352}{28}$

$= \frac{4523}{28}$ cm^2

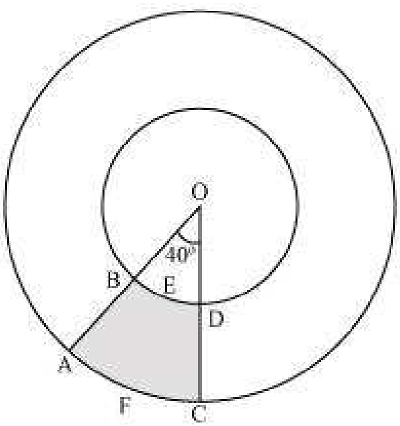
Question 2:

Find the area of the shaded region in the given figure, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40°.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Radius of inner circle = 7 cm

Radius of outer circle = 14 cm

Area of shaded region = Area of sector OAFC − Area of sector OBED

$$= \frac{40^\circ}{360^\circ} \times \pi (14)^2 = \frac{40^\circ}{360^\circ} \times \pi (7)^2$$

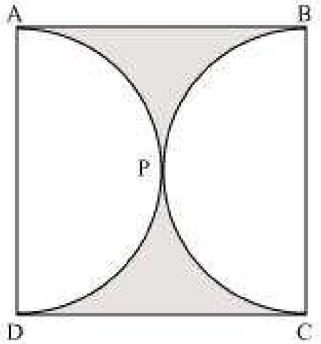
$$= \frac{1}{9} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{9} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{616}{9} - \frac{154}{9} = \frac{462}{9}$$

$$= \frac{154}{3} \text{cm}^2$$

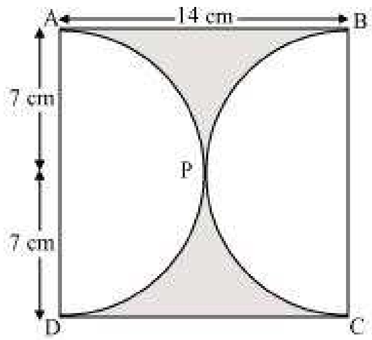
Question 3:

Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

It can be observed from the figure that the radius of each semi-circle is 7 cm.



Area of each semi-circle = $\frac{1}{2}\pi r^2$

$=\frac{1}{2}\times\frac{22}{7}\times(7)^2$

$= 77 \text{ cm}^2$

Area of square ABCD = $(\text{Side})^2 = (14)^2 = 196 \text{ cm}^2$

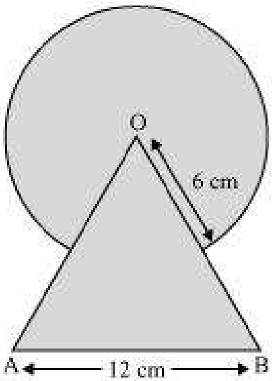
Area of the shaded region

= Area of square ABCD − Area of semi-circle APD − Area of semi-circle BPC

= 196 − 77 − 77 = 196 − 154 = 42 cm^2

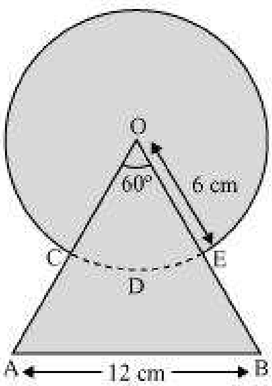
Question 4:

Find the area of the shaded region in the given figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre. $\left[\text{Use } \pi=\frac{22}{7}\right]$



Answer:

We know that each interior angle of an equilateral triangle is of measure 60°.



Area of sector OCDE $= \frac{60^\circ}{360^\circ}\pi r^2$

$= \frac{1}{6}\times\frac{22}{7}\times 6\times 6$

$= \frac{132}{7}\ \text{cm}^2$

Area of $\Delta OAB = \frac{\sqrt{3}}{4}(12)^2 = \frac{\sqrt{3}\times 12\times 12}{4} = 36\sqrt{3}\ \text{cm}^2$

Area of circle = πr^2 $= \frac{22}{7}\times 6\times 6 = \frac{792}{7}\ \text{cm}^2$

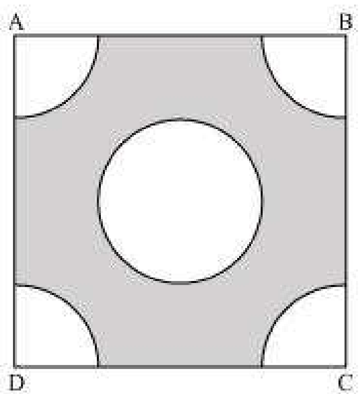
Area of shaded region = Area of ΔOAB + Area of circle − Area of sector OCDE

$$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$$

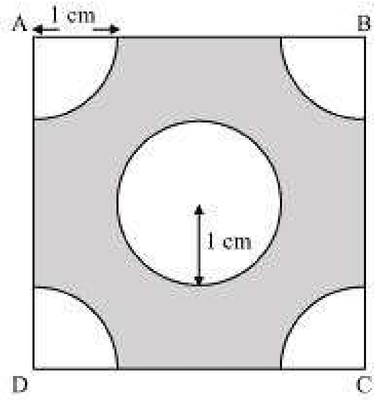
$$= \left(36\sqrt{3} + \frac{660}{7}\right) \text{ cm}^2$$

Question 5:

From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the given figure. Find the area of the remaining portion of the square. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Each quadrant is a sector of 90° in a circle of 1 cm radius.

Area of each quadrant $= \frac{90°}{360°}\pi r^2$

$= \frac{1}{4}\times\frac{22}{7}\times(1)^2 = \frac{22}{28}\text{cm}^2$

Area of square = (Side)^2 = (4)^2 = 16 cm^2

Area of circle = πr^2 = π (1)^2

$= \frac{22}{7}\text{ cm}^2$

Area of the shaded region = Area of square − Area of circle − 4 × Area of quadrant

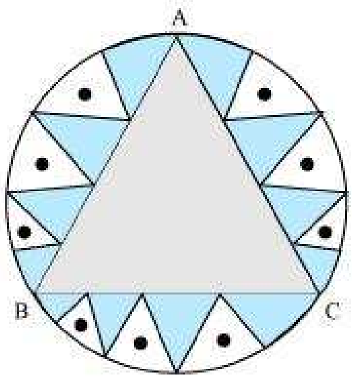
$= 16 - \frac{22}{7} - 4\times\frac{22}{28}$

$= 16 - \frac{22}{7} - \frac{22}{7} = 16 - \frac{44}{7}$

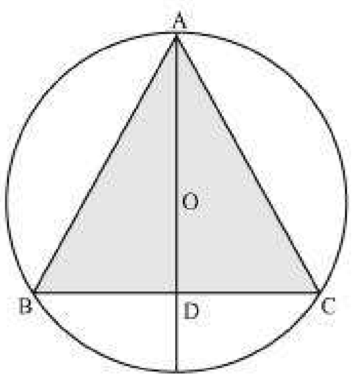
$= \frac{112-44}{7} = \frac{68}{7}\text{ cm}^2$

Question 6:

In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the given figure. Find the area of the design (Shaded region). $\left[\text{Use } \pi = \dfrac{22}{7}\right]$



Answer:



Radius (r) of circle = 32 cm

AD is the median of ΔABC.

$$AO = \frac{2}{3} AD = 32$$

AD = 48 cm

In ΔABD,

$AB^2 = AD^2 + BD^2$

$$AB^2 = (48)^2 + \left(\frac{AB}{2}\right)^2$$

$$\frac{3AB^2}{4} = (48)^2$$

$$AB = \frac{48 \times 2}{\sqrt{3}} = \frac{96}{\sqrt{3}}$$

$$= 32\sqrt{3} \text{ cm}$$

Area of equilateral triangle, $\Delta ABC = \frac{\sqrt{3}}{4}\left(32\sqrt{3}\right)^2$

$$= \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3 = 96 \times 8 \times \sqrt{3}$$

$$= 768\sqrt{3} \text{ cm}^2$$

Area of circle = πr^2

$$= \frac{22}{7} \times (32)^2$$

$$= \frac{22}{7} \times 1024$$

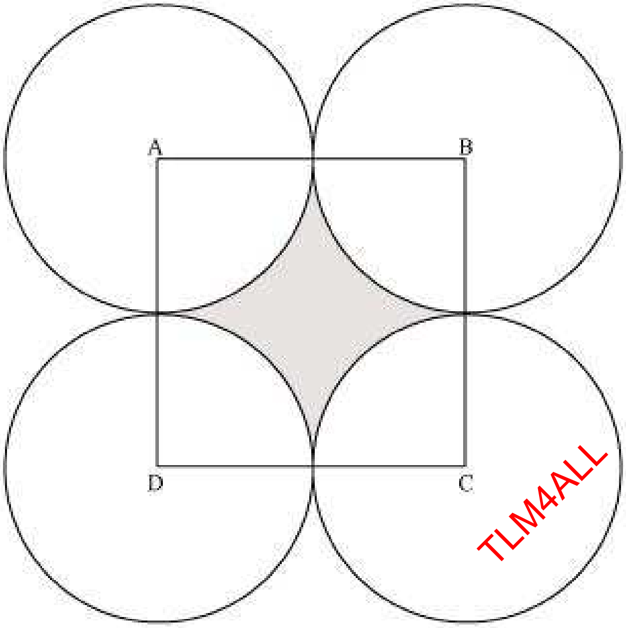
$$= \frac{22528}{7} \text{ cm}^2$$

Area of design = Area of circle − Area of ΔABC

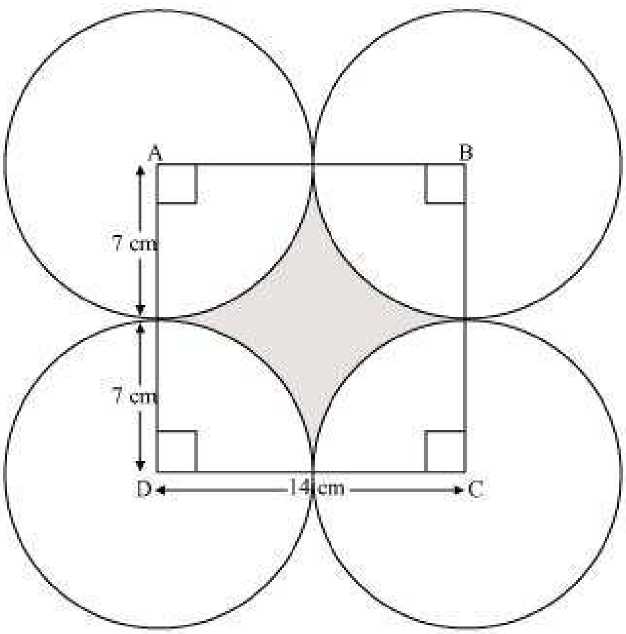
$$= \left(\frac{22528}{7} - 768\sqrt{3}\right) \text{ cm}^2$$

Question 7:

In the given figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Area of each of the 4 sectors is equal to each other and is a sector of 90° in a circle of 7 cm radius.

Area of each sector $= \frac{90°}{360°} \times \pi (7)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

$= \frac{77}{2}\ \text{cm}^2$

Area of square ABCD = $(\text{Side})^2 = (14)^2 = 196\ \text{cm}^2$

Area of shaded portion = Area of square ABCD − 4 × Area of each sector

$= 196 - 4 \times \frac{77}{2} = 196 - 154$

$= 42\ \text{cm}^2$

Therefore, the area of shaded portion is $42\ \text{cm}^2$.

Question 8:

Thegivenfigure depicts a racing track whose left and right ends are semicircular.

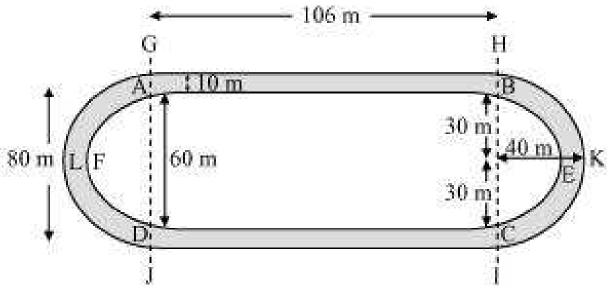
The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) The distance around the track along its inner edge

(ii) The area of the track

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Distance around the track along its inner edge = AB + arc BEC + CD + arc DFA

$$= 106 + \frac{1}{2}\times 2\pi r + 106 + \frac{1}{2}\times 2\pi r$$

$$= 212 + \frac{1}{2}\times 2\times\frac{22}{7}\times 30 + \frac{1}{2}\times 2\times\frac{22}{7}\times 30$$

$$= 212 + 2\times\frac{22}{7}\times 30$$

$$= 212 + \frac{1320}{7}$$

$$= \frac{1484+1320}{7} = \frac{2804}{7}\text{ m}$$

Area of the track = (Area of GHIJ − Area of ABCD) + (Area of semi-circle HKI − Area of semi-circle BEC) + (Area of semi-circle GLJ − Area of semi-circle AFD)

$$= 106\times 80 - 106\times 60 + \frac{1}{2}\times\frac{22}{7}\times(40)^2 - \frac{1}{2}\times\frac{22}{7}\times(30)^2 + \frac{1}{2}\times\frac{22}{7}\times(40)^2 - \frac{1}{2}\times\frac{22}{7}\times(30)^2$$

$$= 106(80-60) + \frac{22}{7}\times(40)^2 - \frac{22}{7}\times(30)^2$$

$$= 106(20) + \frac{22}{7}\left[(40)^2 - (30)^2\right]$$

$$= 2120 + \frac{22}{7}(40-30)(40+30)$$

$$= 2120 + \left(\frac{22}{7}\right)(10)(70)$$

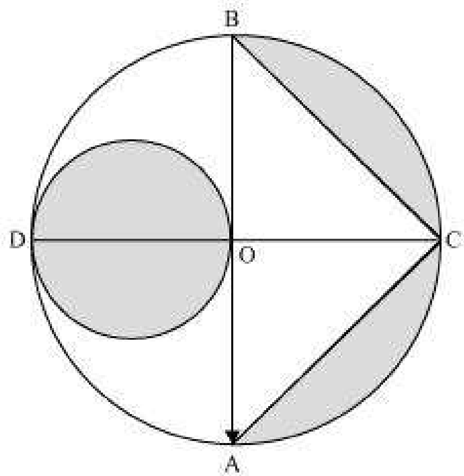
$$= 2120 + 2200$$

$$= 4320\text{ m}^2$$

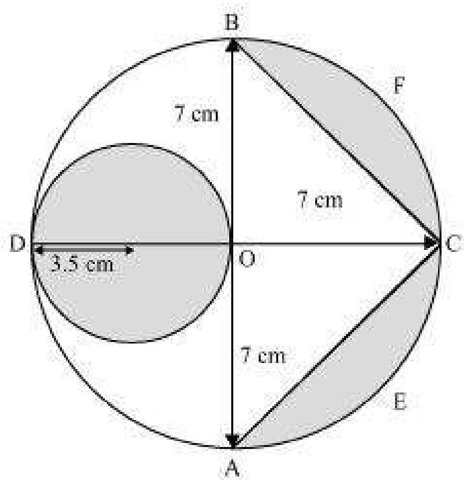
Therefore, the area of the track is 4320 m^2.

Question 9:

In the given figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Radius (r_1) of larger circle = 7 cm

Radius (r_2) of smaller circle $= \frac{7}{2}$ cm

Area of smaller circle $= \pi r_1^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2}$ cm^2

Area of semi-circle AECFB of larger circle $= \frac{1}{2} \pi r_2^2$

$= \frac{1}{2} \times \frac{22}{7} \times (7)^2$

$= 77$ cm^2

Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 14 \times 7 = 49$ cm^2

Area of the shaded region

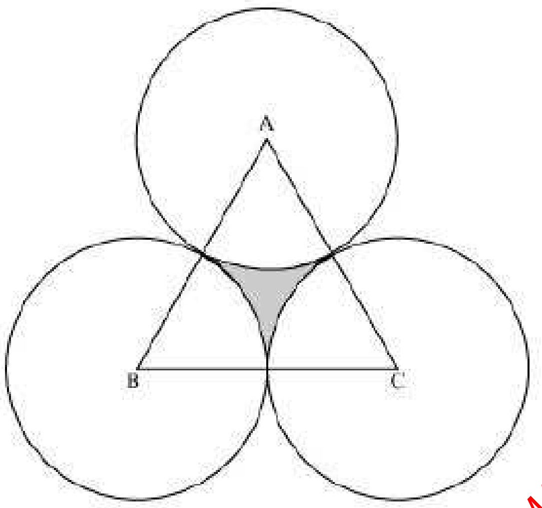
= Area of smaller circle + Area of semi-circle AECFB − Area of ΔABC

$= \frac{77}{2} + 77 - 49$

$= 28 + \frac{77}{2} = 28 + 38.5 = 66.5$ cm^2

Question 10:

The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (See the given figure). Find the area of shaded region. [Use π = 3.14 and $\sqrt{3} = 1.73205$]



Answer:

Let the side of the equilateral triangle be *a*.

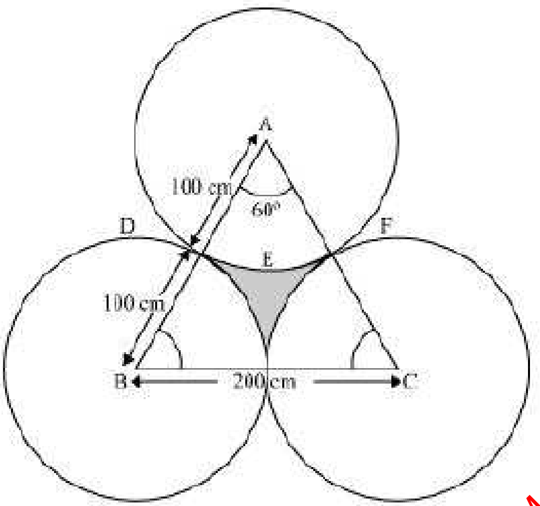
Area of equilateral triangle = 17320.5 cm^2

$$\frac{\sqrt{3}}{4}(a)^2 = 17320.5$$

$$\frac{1.73205}{4}a^2 = 17320.5$$

$$a^2 = 4 \times 10000$$

$$a = 200 \text{ cm}$$



Each sector is of measure 60°.

Area of sector ADEF $= \frac{60°}{360°} \times \pi \times r^2$

$= \frac{1}{6} \times \pi \times (100)^2$

$= \frac{3.14 \times 10000}{6}$

$= \frac{15700}{3}\ \text{cm}^2$

Area of shaded region = Area of equilateral triangle − 3 × Area of each sector

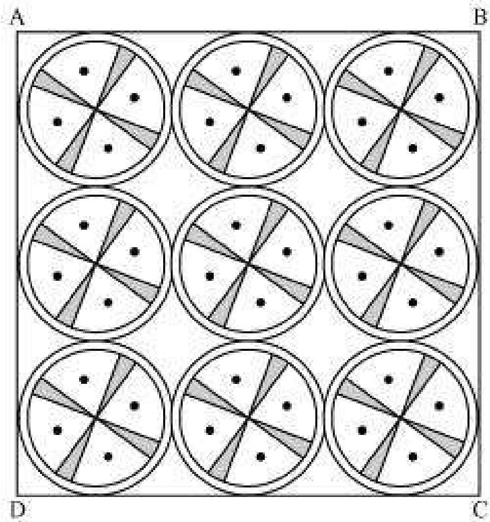
$= 17320.5 - 3 \times \frac{15700}{3}$

$= 17320.5 - 15700 = 1620.5\ \text{cm}^2$

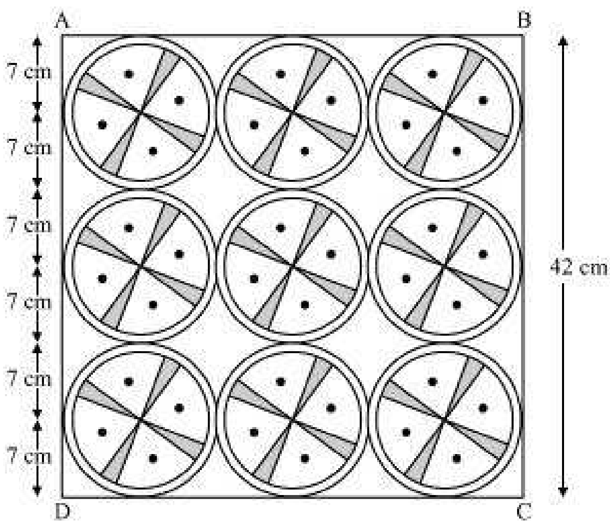
Question 11:

On a square handkerchief, nine circular designs each of radius 7 cm are made (see the given figure). Find the area of the remaining portion of the handkerchief.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



From the figure, it can be observed that the side of the square is 42 cm.

Area of square = $(\text{Side})^2 = (42)^2 = 1764 \text{ cm}^2$

Area of each circle = πr^2 $= \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$

Area of 9 circles = $9 \times 154 = 1386 \text{ cm}^2$

Area of the remaining portion of the handkerchief = 1764 − 1386 = 378 cm^2

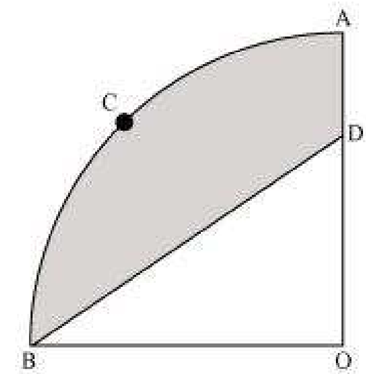
Question 12:

In the given figure, OACB is a quadrant of circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

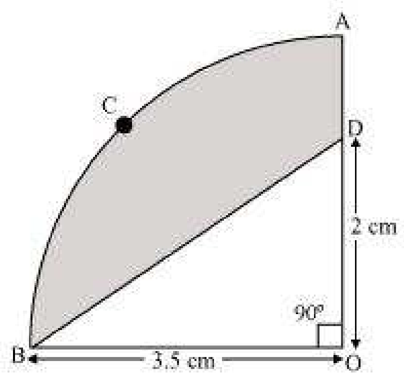
(i) Quadrant OACB

(ii) Shaded region

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$



Answer:



(i) Since OACB is a quadrant, it will subtend 90° angle at O.

Area of quadrant OACB $= \frac{90°}{360°} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= \frac{11 \times 7 \times 7}{2 \times 7 \times 2 \times 2} = \frac{77}{8} \text{ cm}^2$$

(ii) Area of ΔOBD $= \frac{1}{2} \times OB \times OD$

$$= \frac{1}{2} \times 3.5 \times 2$$

$$= \frac{1}{2} \times \frac{7}{2} \times 2$$

$$= \frac{7}{2} \text{ cm}^2$$

Area of the shaded region = Area of quadrant OACB − Area of ΔOBD

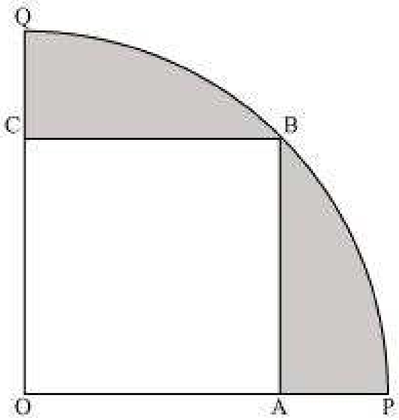
$$= \frac{77}{8} - \frac{7}{2}$$

$$= \frac{77 - 28}{8}$$

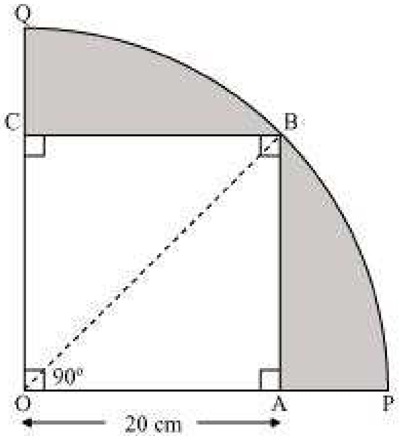
$$= \frac{49}{8} \text{ cm}^2$$

Question 13:

In the given figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. [Use π = 3.14]



Answer:



In ΔOAB,

$OB^2 = OA^2 + AB^2$

$= (20)^2 + (20)^2$

$OB = 20\sqrt{2}$

Radius (r) of circle $= 20\sqrt{2}$ cm

Area of quadrant OPBQ $= \frac{90°}{360°} \times 3.14 \times \left(20\sqrt{2}\right)^2$

$= \frac{1}{4} \times 3.14 \times 800$

$= 628 \text{ cm}^2$

Area of OABC = $(\text{Side})^2 = (20)^2 = 400 \text{ cm}^2$

Area of shaded region = Area of quadrant OPBQ − Area of OABC

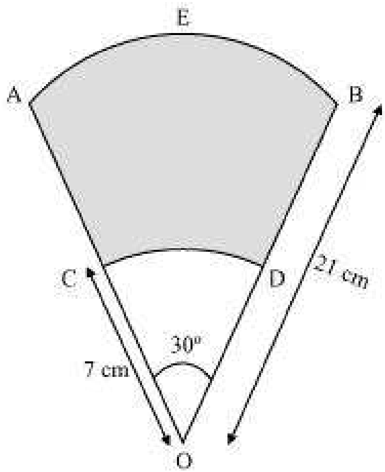
$= (628 - 400) \text{ cm}^2$

$= 228 \text{ cm}^2$

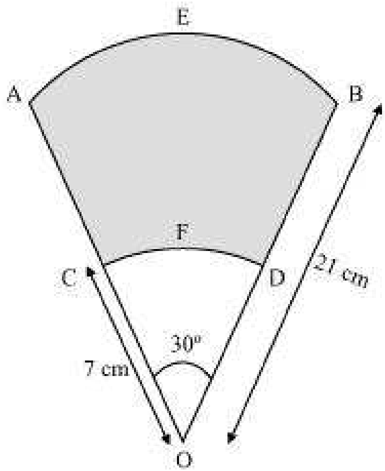
Question 14:

AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see the given figure). If $\angle AOB = 30°$, find the area of the shaded region.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Area of the shaded region = Area of sector OAEB – Area of sector OCFD

$$= \frac{30^\circ}{360^\circ} \times \pi \times (21)^2 - \frac{30^\circ}{360^\circ} \times \pi \times (7)^2$$

$$= \frac{1}{12} \times \pi \left[(21)^2 - (7)^2 \right]$$

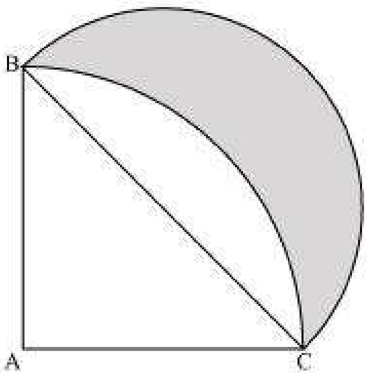
$$= \frac{1}{12} \times \frac{22}{7} \times \left[(21-7)(21+7) \right]$$

$$= \frac{22 \times 14 \times 28}{12 \times 7}$$

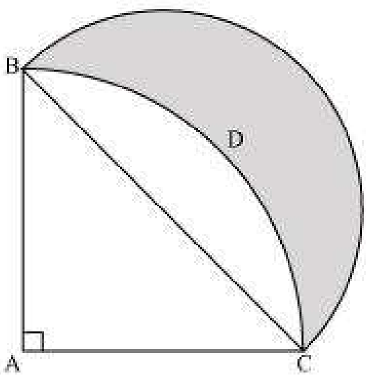
$$= \frac{308}{3} \text{ cm}^2$$

Question 15:

In the given figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7} \right]$



Answer:



As ABC is a quadrant of the circle, $\angle$BAC will be of measure 90º.

In ΔABC,

$BC^2 = AC^2 + AB^2$

$= (14)^2 + (14)^2$

$BC = 14\sqrt{2}$

Radius (r_1) of semi-circle drawn on $BC = \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 14 \times 14$$
$$= 98 \text{ cm}^2$$

Area of sector $\text{ABDC} = \frac{90^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$
$$= 154 \text{ cm}^2$$

$$\text{Area of semi-circle drawn on BC} = \frac{1}{2} \times \pi \times r_1^2 = \frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2$$
$$= \frac{1}{2} \times \frac{22}{7} \times 98 = 154 \text{ cm}^2$$

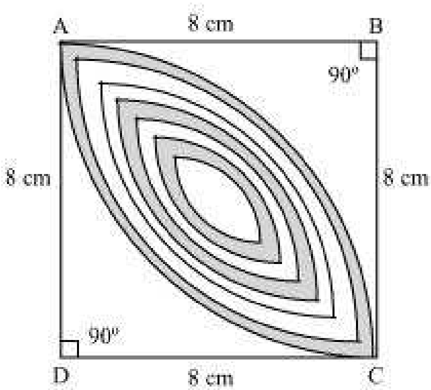
$\text{Area of shaded region} = \text{Area of semi-circle} - \left(\text{Area of sector ABDC} - \text{Area of } \Delta\text{ABC}\right)$ =

154 − (154 − 98)

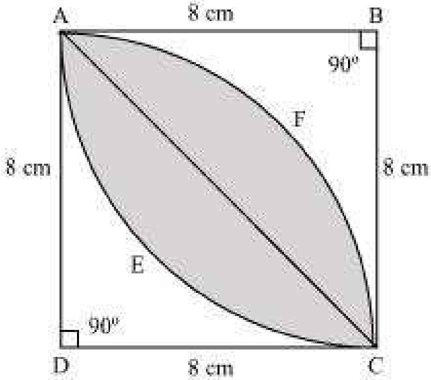
= 98 cm^2

Question 16:

Calculate the area of the designed region in the given figure common between the two quadrants of circles of radius 8 cm each. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



The designed area is the common region between two sectors BAEC and DAFC.

Area of sector $BAEC = \frac{90°}{360°} \times \frac{22}{7} \times (8)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 64$

$= \frac{22 \times 16}{7}$

$= \frac{352}{7} \text{ cm}^2$

Area of ΔBAC $= \frac{1}{2} \times BA \times BC$

$= \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2$

Area of the designed portion = 2 × (Area of segment AEC)

= 2 × (Area of sector BAEC − Area of ΔBAC)

$$= 2\times\left(\frac{352}{7}-32\right) = 2\left(\frac{352-224}{7}\right)$$

$$= \frac{2\times128}{7}$$

$$= \frac{256}{7}\ \text{cm}^2$$

Exercise 13.1

Question 1:

2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboids.

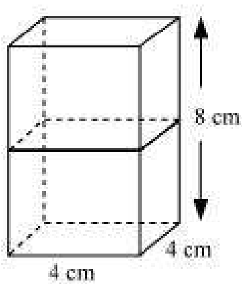
Answer:

Given that,

Volume of cubes = 64 cm^3

$(\text{Edge})^3 = 64$

Edge = 4 cm



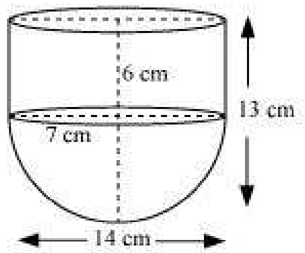
If cubes are joined end to end, the dimensions of the resulting cuboid will be 4 cm, 4 cm, 8 cm.

$$\begin{aligned}\therefore \text{Surface area of cuboids} &= 2(lb + bh + lh)\\ &= 2(4\times4 + 4\times8 + 4\times8)\\ &= 2(16+32+32)\\ &= 2(16+64)\\ &= 2\times80 = 160\text{ cm}^2\end{aligned}$$

Question 2:

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



It can be observed that radius (r) of the cylindrical part and the hemispherical part is the same (i.e., 7 cm).

Height of hemispherical part = Radius = 7 cm

Height of cylindrical part (h) = 13 −7 = 6 cm

Inner surface area of the vessel = CSA of cylindrical part + CSA of hemispherical part

$$= 2\pi rh + 2\pi r^2$$

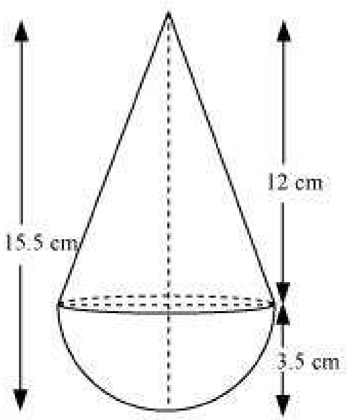
$$\text{Inner surface area of vessel} = 2\times\frac{22}{7}\times 7\times 6 + 2\times\frac{22}{7}\times 7\times 7$$
$$= 44(6+7) = 44\times 13$$
$$= 572 \text{ cm}^2$$

Question 3:

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$

Answer:



It can be observed that the radius of the conical part and the hemispherical part is same (i.e., 3.5 cm).

Height of hemispherical part = Radius (r) = 3.5 = $\frac{7}{2}$ cm

Height of conical part (h) = 15.5 −3.5 = 12 cm

Slant height (l) of conical part $= \sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{7}{2}\right)^2 + (12)^2} = \sqrt{\frac{49}{4} + 144} = \sqrt{\frac{49 + 576}{4}}$$

$$= \sqrt{\frac{625}{4}} = \frac{25}{2}$$

Total surface area of toy = CSA of conical part + CSA of hemispherical part

$= \pi r l + 2\pi r^2$

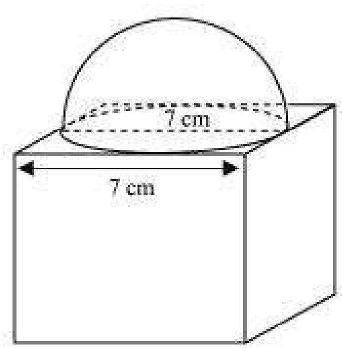
$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 137.5 + 77 = 214.5 \text{cm}^2$

Question 4:

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



From the figure, it can be observed that the greatest diameter possible for such hemisphere is equal to the cube's edge, i.e., 7cm.

Radius (r) of hemispherical part = $\frac{7}{2}$ = 3.5cm

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part − Area of base of hemispherical part

= 6 (Edge)^2 $+ 2\pi r^2 - \pi r^2$ = 6 (Edge)^2 $+ \pi r^2$

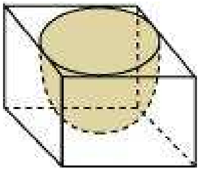
$$\text{Total surface area of solid} = 6(7)^2 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 294 + 38.5 = 332.5\,\text{cm}^2$$

Question 5:

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter *l* of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer:



Diameter of hemisphere = Edge of cube = l

Radius of hemisphere = $\frac{l}{2}$

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part − Area of base of hemispherical part

= 6 (Edge)^2 $+ 2\pi r^2 - \pi r^2$ = 6 (Edge)^2 + πr^2

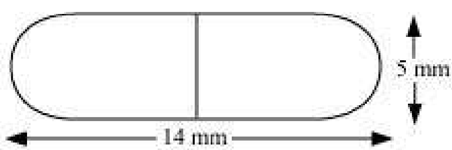
$$\text{Total surface area of solid } = 6l^2 + \pi \times \left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \frac{\pi l^2}{4}$$

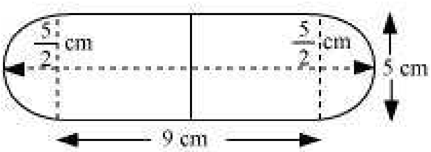
$$= \frac{1}{4}(24+\pi)l^2 \text{ unit}^2$$

Question 6:

A medicine capsule is in the shape of cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part

$$= \frac{\text{Diameter of the capsule}}{2} = \frac{5}{2}$$

Length of cylindrical part (h) = Length of the entire capsule − 2 × r

= 14 − 5 = 9 cm

Surface area of capsule = 2×CSA of hemispherical part + CSA of cylindrical part

$$= 2 \times 2\pi r^2 + 2\pi rh$$

$$= 4\pi\left(\frac{5}{2}\right)^2 + 2\pi\left(\frac{5}{2}\right)(9)$$

$$= 25\pi + 45\pi$$

$$= 70\pi \text{ mm}^2$$

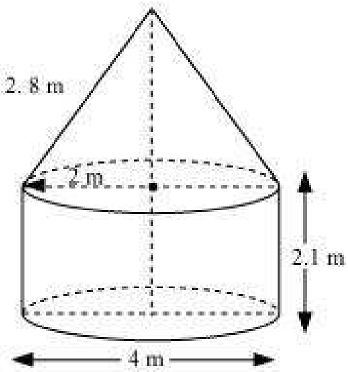
$$= 70 \times \frac{22}{7}$$

$$= 220 \text{ mm}^2$$

Question 7:

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.) $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Given that,

Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

Radius of the cylindrical part = 2 m

Slant height (l) of conical part = 2.8 m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi[2.8 + 2 \times 2.1] = 2\pi[2.8 + 4.2] = 2 \times \frac{22}{7} \times 7$

$= 44 \text{ m}^2$

Cost of 1 m^2 canvas = Rs 500

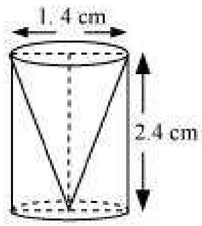
Cost of 44 m^2 canvas = 44 × 500 = 22000

Therefore, it will cost Rs 22000 for making such a tent.

Question 8:

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Given that,

Height (h) of the conical part = Height (h) of the cylindrical part = 2.4 cm

Diameter of the cylindrical part = 1.4 cm

Therefore, radius (r) of the cylindrical part = 0.7 cm

$$\text{Slant height } (l) \text{ of conical part} = \sqrt{r^2 + h^2}$$
$$= \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76}$$
$$= \sqrt{6.25} = 2.5$$

Total surface area of the remaining solid will be

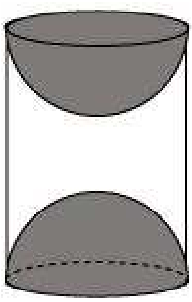
= CSA of cylindrical part + CSA of conical part + Area of cylindrical base

$$= 2\pi rh + \pi rl + \pi r^2$$
$$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times 0.7 \times 2.5 + \frac{22}{7} \times 0.7 \times 0.7$$
$$= 4.4 \times 2.4 + 2.2 \times 2.5 + 2.2 \times 0.7$$
$$= 10.56 + 5.50 + 1.54 = 17.60 \text{ cm}^2$$

The total surface area of the remaining solid to the nearest cm^2 is 18 cm^2

Question 9:

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in given figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:

Given that,

Radius (r) of cylindrical part = Radius (r) of hemispherical part = 3.5 cm

Height of cylindrical part (h) = 10 cm

Surface area of article = CSA of cylindrical part + 2 × CSA of hemispherical part

$$= 2\pi rh + 2 \times 2\pi r^2$$

$$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5$$

$$= 70\pi + 49\pi$$

$$= 119\pi$$

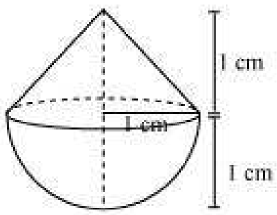
$$= 17 \times 22 = 374 \text{ cm}^2$$

Exercise 13.2

Question 1:

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Answer:



Given that,

Height (h) of conical part = Radius(r) of conical part = 1 cm

Radius(r) of hemispherical part = Radius of conical part (r) = 1 cm

Volume of solid = Volume of conical part + Volume of hemispherical part

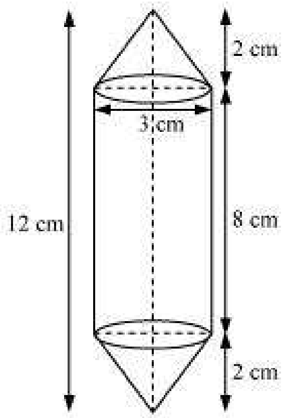
$$=\frac{1}{3}\pi r^2 h+\frac{2}{3}\pi r^3$$

$$=\frac{1}{3}\pi(1)^2(1)+\frac{2}{3}\pi(1)^3=\frac{\pi}{3}+\frac{2\pi}{3}=\pi\text{ cm}^3$$

Question 2:

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm. if each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.) $\left[\text{Use } \pi=\frac{22}{7}\right]$

Answer:



From the figure, it can be observed that

Height (h_1) of each conical part = 2 cm

Height (h_2) of cylindrical part = 12 − 2 × Height of conical part

= 12 − 2 ×2 = 8 cm

Radius (r) of cylindrical part = Radius of conical part = $\frac{3}{2}$ cm

Volume of air present in the model = Volume of cylinder + 2 × Volume of cones

$$= \pi r^2 h_2 + 2 \times \frac{1}{3} \pi r^2 h_1$$

$$= \pi\left(\frac{3}{2}\right)^2 (8) + 2 \times \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 (2)$$

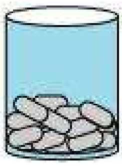
$$= \pi \times \frac{9}{4} \times 8 + \frac{2}{3}\pi \times \frac{9}{4} \times 2$$

$$= 18\pi + 3\pi = 21\pi = 66 \text{ cm}^2$$

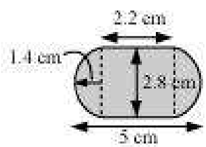
Question 3:

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see the given figure).

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part = $\frac{2.8}{2} = 1.4\text{ cm}$

Length of each hemispherical part = Radius of hemispherical part = 1.4 cm

Length (h) of cylindrical part = 5 − 2 × Length of hemispherical part

= 5 − 2 × 1.4 = 2.2 cm

Volume of one gulab jamun = Vol. of cylindrical part + 2 × Vol. of hemispherical part

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (1.4)^2 \times 2.2 + \frac{4}{3}\pi(1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$$= 13.552 + 11.498 = 25.05\text{ cm}^3$$

Volume of 45 gulab jamuns = 45×25.05 = 1,127.25 cm^3

Volume of sugar syrup = 30% of volume

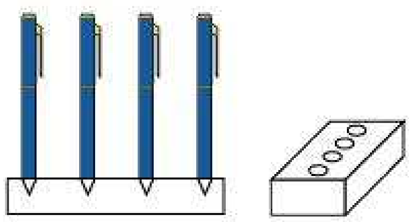
$= \frac{30}{100} \times 1,127.25$

$= 338.17 \text{ cm}^3$

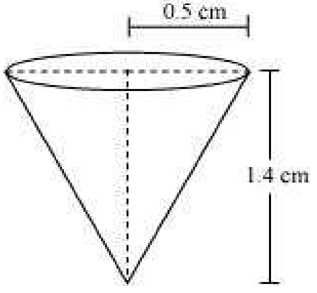
$\simeq 338 \text{ cm}^3$

Question 4:

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboids are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see the following figure). $\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Depth (h) of each conical depression = 1.4 cm

Radius (r) of each conical depression = 0.5 cm

Volume of wood = Volume of cuboid − 4 × Volume of cones

$=\text{lbh}-4\times\frac{1}{3}\pi r^2 h$

$=15\times10\times3.5-4\times\frac{1}{3}\times\frac{22}{7}\times\left(\frac{1}{2}\right)^2\times1.4$

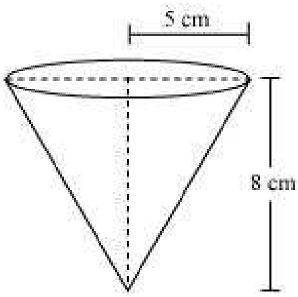
$=525-1.47$

$=523.53\text{ cm}^3$

Question 5:

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer:



Height (h) of conical vessel = 8 cm

Radius (r_1) of conical vessel = 5 cm

Radius (r_2) of lead shots = 0.5 cm

Let n number of lead shots were dropped in the vessel.

Volume of water spilled = Volume of dropped lead shots

$$\frac{1}{4}\times \text{Volume of cone} = n\times\frac{4}{3}\ \ r_2^3$$

$$\frac{1}{4}\times\frac{1}{3}\pi r_1^2 h = n\times\frac{4}{3}\pi r_2^3$$

$$r_1^2 h = n\times 16 r_2^3$$

$$5^2\times 8 = n\times 16\times(0.5)^3$$

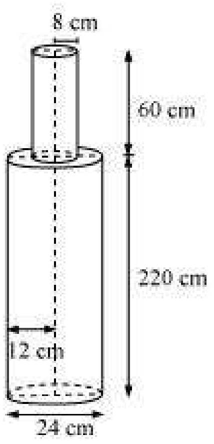
$$n=\frac{25\times 8}{16\times\left(\frac{1}{2}\right)^3}=100$$

Hence, the number of lead shots dropped in the vessel is 100.

Question 6:

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. [Use π = 3.14]

Answer:



From the figure, it can be observed that

Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = $\frac{24}{2}$ = 12 cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of smaller cylinder = 8 cm

$$\text{Total volume of pole} = \text{Volume of larger cylinder} + \text{Volume of smaller cylinder}$$
$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$
$$= \pi (12)^2 \times 220 + \pi (8)^2 \times 60$$
$$= \pi [144 \times 220 + 64 \times 60]$$
$$= 35520 \times 3.14 = 1,11,532.8 \text{ cm}^3$$

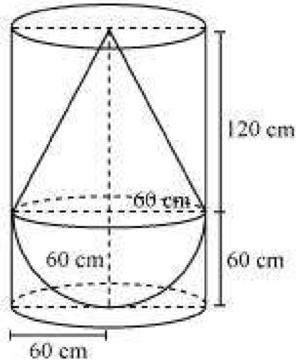
Mass of 1 cm^3 iron = 8 g

Mass of 111532.8 cm^3 iron = 111532.8 × 8 = 892262.4 g = 892.262 kg

Question 7:

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Radius (r) of hemispherical part = Radius (r) of conical part = 60 cm

Height (h_2) of conical part of solid = 120 cm

Height (h_1) of cylinder = 180 cm

Radius (r) of cylinder = 60 cm

Volume of water left = Volume of cylinder − Volume of solid

$$= \text{Volume of cylinder } - (\text{Volume of cone} + \text{Volume of hemisphere})$$

$$= \pi r^2 h_1 - \left(\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3 \right)$$

$$= \pi(60)^2 (180) - \left(\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3 \right)$$

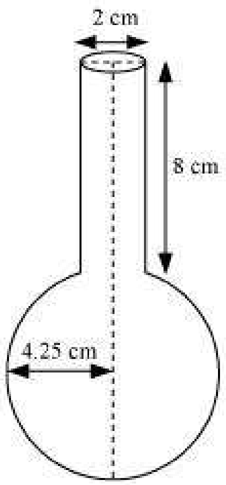
$$= \pi(60)^2 \left[(180) - (40+40) \right]$$

$$= \pi(3,600)(100) = 3,60,000\pi \text{ cm}^3 = 1131428.57 \text{ cm}^3 = 1.131 \text{ m}^3$$

Question 8:

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter o the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Answer:



Height (h) of cylindrical part = 8 cm

Radius (r_2) of cylindrical part = $\frac{2}{2}=1$ cm

Radius (r_1) spherical part = $\frac{8.5}{2}=4.25$ cm

Volume of vessel = Volume of sphere + Volume of cylinder

$$=\frac{4}{3}\pi r_1^3+\pi r_2^2 h$$

$$=\frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3+\pi(1)^2(8)$$

$$=\frac{4}{3}\times 3.14\times 76.765625+8\times 3.14$$

$$=321.392+25.12$$

$$=346.512$$

$$=346.51 \text{ cm}^3$$

Hence, she is wrong.

Exercise 13.3

Question 1:

A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Answer:

Radius (r_1) of hemisphere = 4.2 cm

Radius (r_2) of cylinder = 6 cm

Let the height of the cylinder be h.

The object formed by recasting the hemisphere will be the same in volume.

Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\frac{4}{3}\pi(4.2)^3 = \pi(6)^2 h$$

$$\frac{4}{3}\times\frac{4.2\times4.2\times4.2}{36} = h$$

$$h = (1.4)^3 = 2.74 \text{ cm}$$

Hence, the height of the cylinder so formed will be 2.74 cm.

Question 2:

Metallic spheres of radii 6 cm, 8 cm, and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Answer:

Radius (r_1) of 1st sphere = 6 cm

Radius (r_2) of 2nd sphere = 8 cm

Radius (r_3) of 3rd sphere = 10 cm

Let the radius of the resulting sphere be r.

The object formed by recasting these spheres will be same in volume as the sum of the volumes of these spheres.

Volume of 3 spheres = Volume of resulting sphere

$$\frac{4}{3}\pi\left[r_1^3+r_2^3+r_3^3\right]=\frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi\left[6^3+8^3+10^3\right]=\frac{4}{3}\pi r^3$$

$$r^3=216+512+1000=1728$$

$$r=12 \text{ cm}$$

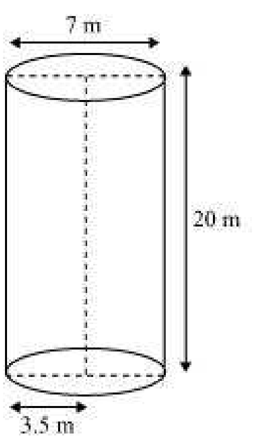
Therefore, the radius of the sphere so formed will be 12 cm.

Question 3:

A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

$$\left[\text{Use } \pi=\frac{22}{7}\right]$$

Answer:



The shape of the well will be cylindrical.

Depth (h) of well = 20 m

Radius (r) of circular end of well = $\frac{7}{2}$ m

Area of platform = Length × Breadth = 22 × 14 m^2

Let height of the platform = H

Volume of soil dug from the well will be equal to the volume of soil scattered on the platform.

Volume of soil from well = Volume of soil used to make such platform

$$\pi \times r^2 \times h = \text{Area of platform} \times \text{Height of platform}$$

$$\pi \times \left(\frac{7}{2}\right)^2 \times 20 = 22 \times 14 \times H$$

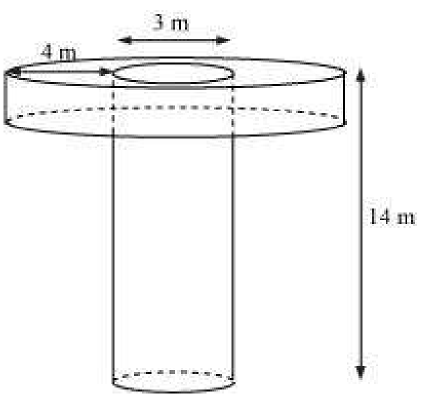
$$\therefore H = \frac{22}{7} \times \frac{49}{4} \times \frac{20}{22 \times 14} = \frac{5}{2} \text{ m} = 2.5 \text{ m}$$

Therefore, the height of such platform will be 2.5 m.

Question 4:

A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Answer:



The shape of the well will be cylindrical.

Depth (h_1) of well = 14 m

Radius (r_1) of the circular end of well = $\frac{3}{2}$ m

Width of embankment = 4 m

From the figure, it can be observed that our embankment will be in a cylindrical shape having outer radius (r_2) as $4+\frac{3}{2}=\frac{11}{2}$ m and inner radius (r_1) as $\frac{3}{2}$ m.

Let the height of embankment be h_2.

Volume of soil dug from well = Volume of earth used to form embankment

$$\pi \times r_1^2 \times h_1 = \pi \times \left(r_2^2 - r_1^2\right) \times h_2$$

$$\pi \times \left(\frac{3}{2}\right)^2 \times 14 = \pi \times \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \times h$$

$$\frac{9}{4} \times 14 = \frac{112}{4} \times h$$

$$h = \frac{9}{8} = 1.125 \text{ m}$$

Therefore, the height of the embankment will be 1.125 m.

Question 5:

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Answer:

Height (h_1) of cylindrical container = 15 cm

Radius (r_1) of circular end of container = $\frac{12}{2} = 6$ cm

Radius (r_2) of circular end of ice-cream cone = $\frac{6}{2} = 3$ cm

Height (h_2) of conical part of ice-cream cone = 12 cm

Let n ice-cream cones be filled with ice-cream of the container.

Volume of ice-cream in cylinder = n × (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi r_1^2 h_1 = n\left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3\right)$$

$$n = \frac{6^2 \times 15}{\frac{1}{3}\times 9\times 12 + \frac{2}{3}\times (3)^3}$$

$$n = \frac{36\times 15\times 3}{108+54}$$

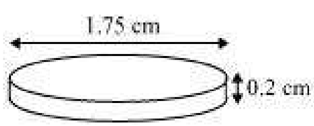
$$n = 10$$

Therefore, 10 ice-cream cones can be filled with the ice-cream in the container.

Question 6:

How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions $5.5\text{ cm}\times 10\text{ cm}\times 3.5\text{ cm}$? $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



Coins are cylindrical in shape.

Height (h_1) of cylindrical coins = 2 mm = 0.2 cm

Radius (r) of circular end of coins = $\frac{1.75}{2} = 0.875\text{ cm}$

Let n coins be melted to form the required cuboids.

Volume of n coins = Volume of cuboids

$$n\times\pi\times r^2\times h_1 = l\times b\times h$$

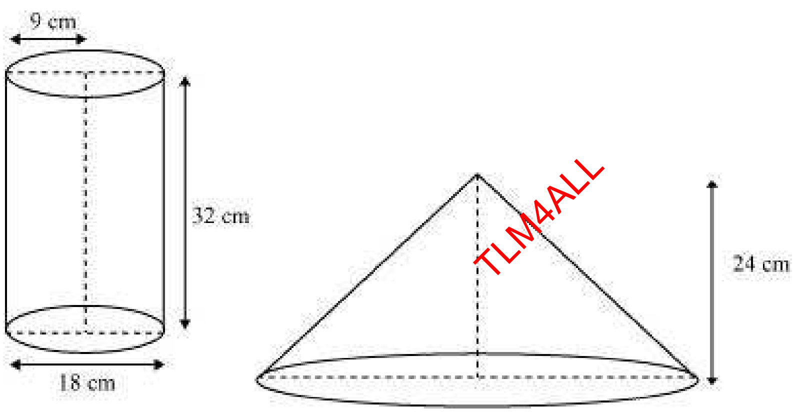
$$n\times\pi\times(0.875)^2\times 0.2 = 5.5\times 10\times 3.5$$

$$n = \frac{5.5\times 10\times 3.5\times 7}{(0.875)^2\times 0.2\times 22} = 400$$

Question 7:

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. Find the radius and slant height of the heap.

Answer:



Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let the radius of the circular end of conical heap be r_2.

The volume of sand in the cylindrical bucket will be equal to the volume of sand in the conical heap.

Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$$\pi \times r_1^2 \times h_1 = \frac{1}{3}\pi \times r_2^2 \times h_2$$

$$\pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$$

$$\pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$$

$$r_2^2 = \frac{3 \times 18^2 \times 32}{24} = 18^2 \times 4$$

$r_2 = 18 \times 2 = 36$ cm

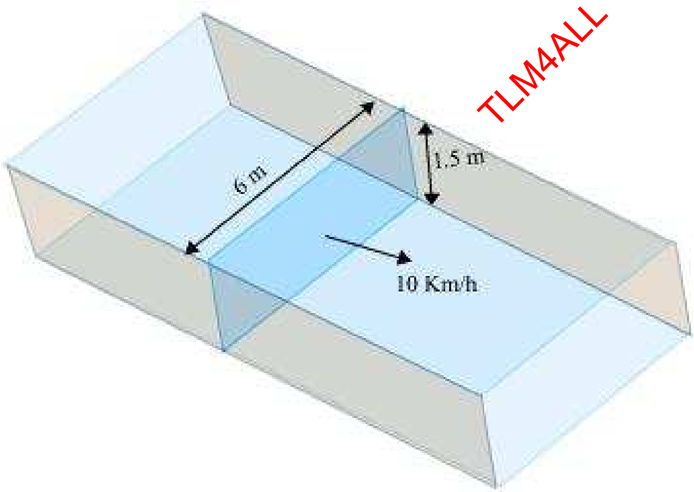
Slant height = $\sqrt{36^2 + 24^2} = \sqrt{12^2 \times (3^2 + 2^2)} = 12\sqrt{13}$ cm

Therefore, the radius and slant height of the conical heap are 36 cm and $12\sqrt{13}$ cm respectively

Question 8:

Water in canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. how much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Answer:



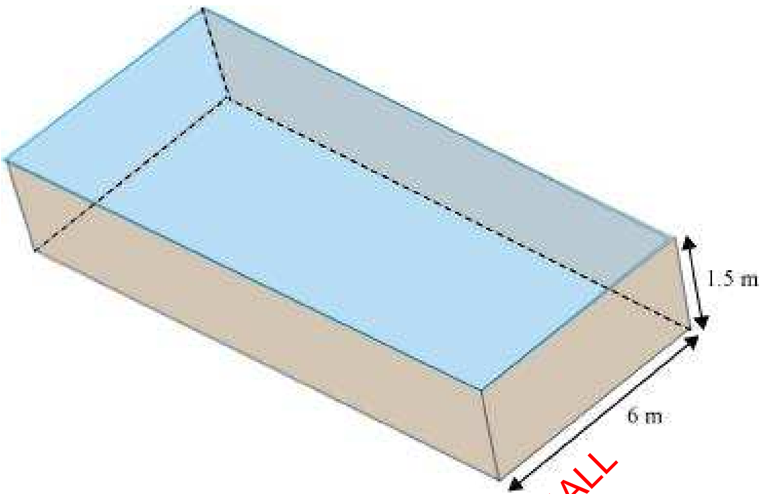
Consider an area of cross-section of canal as ABCD.

Area of cross-section = $6 \times 1.5 = 9$ m^2

Speed of water = 10 km/h = $\frac{10000}{60}$ metre/min

Volume of water that flows in 1 minute from canal = $9 \times \frac{10000}{60}$ =1500 m^3

Volume of water that flows in 30 minutes from canal = $30 \times 1500 = 45000$ m^3



Let the irrigated area be A. Volume of water irrigating the required area will be equal to the volume of water that flowed in 30 minutes from the canal.

Vol. of water flowing in 30 minutes from canal = Vol. of water irrigating the reqd. area

$$45000 = \frac{A \times 8}{100}$$

A = 562500 m^2

Therefore, area irrigated in 30 minutes is 562500 m^2.

Question 9:

A farmer connects a pipe of internal diameter 20 cm form a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Answer:

Consider an area of cross-section of pipe as shown in the figure.

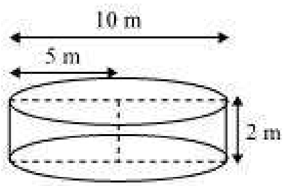
Radius (r_1) of circular end of pipe = $\frac{20}{200} = 0.1 \text{m}$

Area of cross-section = $\pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi \text{ m}^2$

Speed of water = 3 km/h = $\frac{3000}{60} = 50 \text{ metre/min}$

Volume of water that flows in 1 minute from pipe = $50 \times 0.01\pi = 0.5\pi \text{ m}^3$

Volume of water that flows in t minutes from pipe = $t \times 0.5\pi \text{ m}^3$



Radius (r_2) of circular end of cylindrical tank = $\frac{10}{2} = 5$ m

Depth (h_2) of cylindrical tank = 2 m

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe = Volume of water in tank

$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$

$t \times 0.5 = 5^2 \times 2$

$t = 100$

Therefore, the cylindrical tank will be filled in 100 minutes.

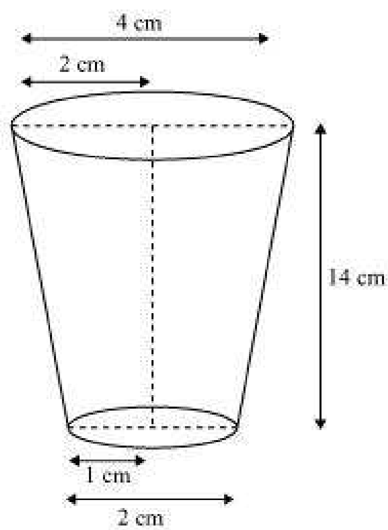
Exercise 13.4

Question 1:

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$

Answer:



Radius (r_1) of upper base of glass $= \frac{4}{2} = 2$ cm

Radius (r_2) of lower base of glass $= \frac{2}{2} = 1$ cm

Capacity of glass = Volume of frustum of cone

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3}\pi h\left[(2)^2 + (1)^2 + (2)(1)\right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14\left[4+1+2\right]$$

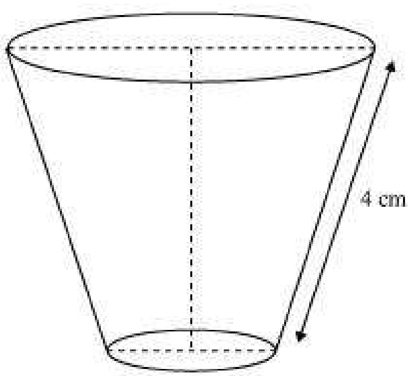
$$= \frac{308}{3} = 102\frac{2}{3}\text{ cm}^3$$

Therefore, the capacity of the glass is $102\frac{2}{3}\text{ cm}^3$.

Question 2:

The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. find the curved surface area of the frustum.

Answer:



Perimeter of upper circular end of frustum = 18

$2\pi r_1 = 18$

$$r_1 = \frac{9}{\pi}$$

Perimeter of lower end of frustum = 6 cm

$2\pi r_2 = 6$

$r_2 = \frac{3}{\pi}$

Slant height (l) of frustum = 4

CSA of frustum = π (r_1 + r_2) l

$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right)4$

$= 12 \times 4$

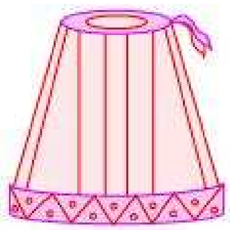
$= 48\,\text{cm}^2$

Therefore, the curved surface area of the frustum is 48 cm^2.

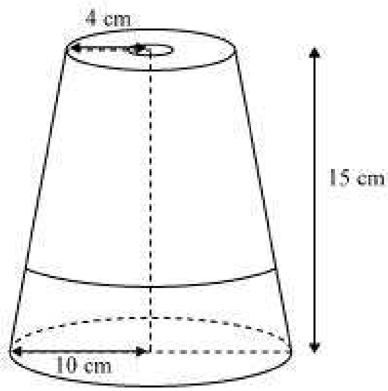
Question 3:

A *fez*, the cap used by the Turks, is shaped like the frustum of a cone (see the figure given below). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material use for making it.

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Answer:



Radius (r_2) at upper circular end = 4 cm

Radius (r_1) at lower circular end = 10 cm

Slant height (l) of frustum = 15 cm

Area of material used for making the fez = CSA of frustum + Area of upper circular end

$$= \pi\,(r_1 + r_2)\,l + \pi r_2^2$$

$= \pi\ (10 + 4)\ 15 + \pi\ (4)^2$

$= \pi\ (14)\ 15 + 16\ \pi$

$$= 210\pi + 16\pi = \frac{226 \times 22}{7}$$

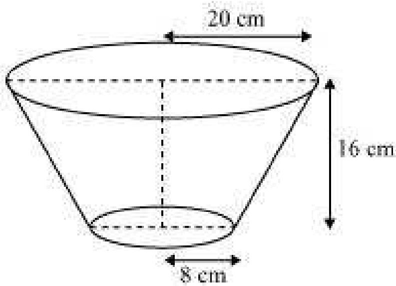
$$= 710\frac{2}{7}\text{cm}^2$$

Therefore, the area of material used for making it is $710\frac{2}{7}\text{cm}^2$.

Question 4:

A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and at the rate of Rs.20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs.8 per 100 cm^2. [Take π = 3.14]

Answer:



Radius (r_1) of upper end of container = 20 cm

Radius (r_2) of lower end of container = 8 cm

Height (h) of container = 16 cm

Slant height (l) of frustum = $\sqrt{(r_1 - r_2)^2 + h^2}$

$$= \sqrt{(20-8)^2 + (16)^2}$$

$$= \sqrt{(12)^2 + (16)^2} = \sqrt{144 + 256}$$

$$= 20 \text{ cm}$$

Capacity of container = Volume of frustum

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3} \times 3.14 \times 16 \times \left[(20)^2 + (8)^2 + (20)(8)\right]$$

$$= \frac{1}{3} \times 3.14 \times 16(400 + 64 + 160)$$

$$= \frac{1}{3} \times 3.14 \times 16 \times 624$$

$$= 10449.92 \text{ cm}^3$$

$$= 10.45 \text{ litres.}$$

Cost of 1 litre milk = Rs 20

Cost of 10.45 litre milk = 10.45 × 20

= Rs 209

Area of metal sheet used to make the container

$= \pi\left(r_1 + r_2\right)l + \pi r_2^2$

= π (20 + 8) 20 + π $(8)^2$

= 560 π + 64 π = 624 π cm^2

Cost of 100 cm^2 metal sheet = Rs 8

$$\text{Cost of } 624\ \pi\ \text{cm}^2 \text{ metal sheet} = \frac{624 \times 3.14 \times 8}{100}$$
$$= 156.75$$

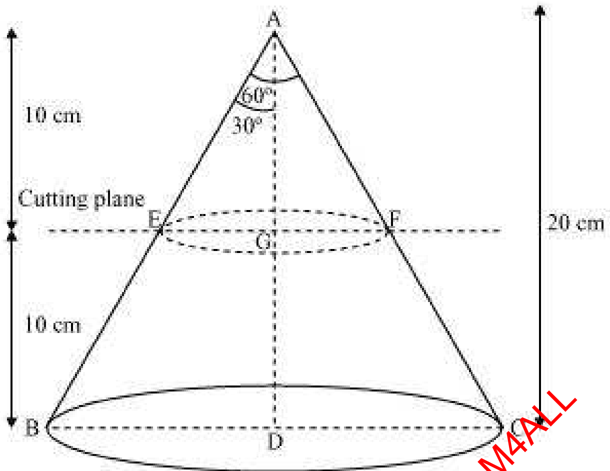
Therefore, the cost of the milk which can completely fill the container is Rs 209 and the cost of metal sheet used to make the container is Rs 156.75.

Question 5:

A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained is drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer:



In ΔAEG,

$$\frac{EG}{AG} = \tan 30°$$

$$EG = \frac{10}{\sqrt{3}}\text{cm} = \frac{10\sqrt{3}}{3}$$

In ΔABD,

$$\frac{BD}{AD} = \tan 30°$$

$$BD = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}\text{cm}$$

Radius (r_1) of upper end of frustum = $\frac{10\sqrt{3}}{3}$ cm

Radius (r_2) of lower end of container = $\frac{20\sqrt{3}}{3}$ cm

Height (h) of container = 10 cm

Volume of frustum $= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$=\frac{1}{3}\times\pi\times10\left[\left(\frac{10\sqrt{3}}{3}\right)^2+\left(\frac{20\sqrt{3}}{3}\right)^2+\frac{\left(10\sqrt{3}\right)\left(20\sqrt{3}\right)}{3\times3}\right]$$

$$=\frac{10}{3}\pi\left[\frac{100}{3}+\frac{400}{3}+\frac{200}{3}\right]$$

$$=\frac{10}{3}\times\frac{22}{7}\times\frac{700}{3}=\frac{22000}{9}\text{cm}^3$$

Radius (r) of wire = $\frac{1}{16}\times\frac{1}{2}=\frac{1}{32}\text{cm}$

Let the length of wire be l.

Volume of wire = Area of cross-section × Length

= (πr^2) (l)

$$=\pi\left(\frac{1}{32}\right)^2\times l$$

Volume of frustum = Volume of wire

$$\frac{22000}{9}=\frac{22}{7}\times\left(\frac{1}{32}\right)^2\times l$$

$$\frac{7000}{9}\times1024=l$$

$$l=796444.44\text{ cm}$$

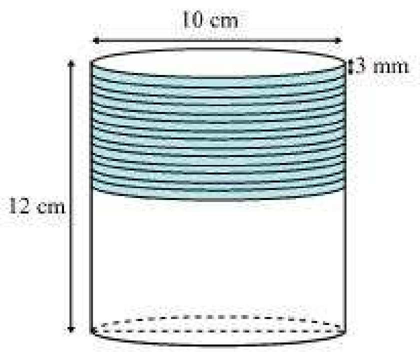
$$=7964.44\text{ metres}$$

Exercise 13.5

Question 1:

A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Answer:



It can be observed that 1 round of wire will cover 3 mm height of cylinder.

$$\text{Number of rounds} = \frac{\text{Height of cylinder}}{\text{Diameter of wire}}$$

$$= \frac{12}{0.3} = 40 \text{ rounds}$$

Length of wire required in 1 round = Circumference of base of cylinder

$= 2\pi r = 2\pi \times 5 = 10\pi$

Length of wire in 40 rounds = $40 \times 10\pi$

$$= \frac{400 \times 22}{7} = \frac{8800}{7}$$

= 1257.14 cm = 12.57 m

Radius of wire $= \frac{0.3}{2} = 0.15 \text{ cm}$

Volume of wire = Area of cross-section of wire × Length of wire

$= \pi(0.15)^2 \times 1257.14$

$= 88.898 \text{ cm}^3$

Mass = Volume × Density

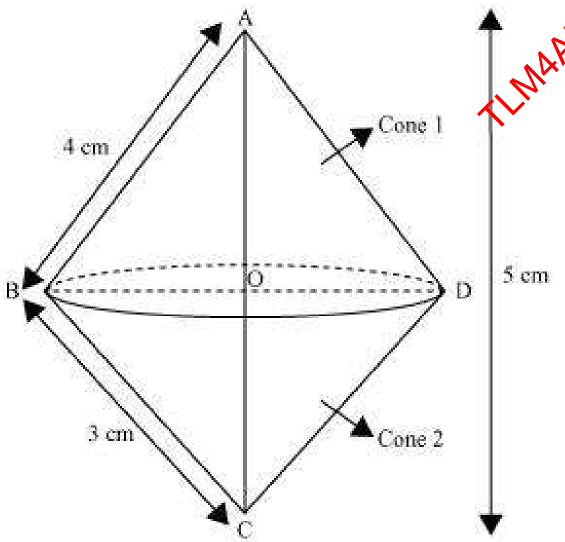
= 88.898 × 8.88

= 789.41 gm

Question 2:

A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)

Answer:



The double cone so formed by revolving this right-angled triangle ABC about its hypotenuse is shown in the figure.

Hypotenuse $AC = \sqrt{3^2 + 4^2}$

$= \sqrt{25}$ = 5 cm

Area of ΔABC $= \frac{1}{2} \times AB \times AC$

$\frac{1}{2}\times AC\times OB=\frac{1}{2}\times 4\times 3$

$\frac{1}{2}\times 5\times OB=6$

$OB=\frac{12}{5}=2.4\text{ cm}$

Volume of double cone = Volume of cone 1 + Volume of cone 2

$=\frac{1}{3}\pi r^2 h_1+\frac{1}{3}\pi r^2 h_2$

$=\frac{1}{3}\pi r^2(h_1+h_2)=\frac{1}{3}\pi r^2(OA+OC)$

$=\frac{1}{3}\times 3.14\times(2.4)^2(5)$

= 30.14 cm^3

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

$= \pi r l_1 + \pi r l_2$

$=\pi r[4+3]=3.14\times 2.4\times 7$

= 52.75 cm^2

Question 3:

A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Answer:

Volume of cistern = 150 × 120 × 110

= 1980000 cm^3

Volume to be filled in cistern = 1980000 − 129600

= 1850400 cm^3

Let n numbers of porous bricks were placed in the cistern.

Volume of n bricks = $n \times 22.5 \times 7.5 \times 6.5$

$= 1096.875n$

As each brick absorbs one-seventeenth of its volume, therefore, volume absorbed by these bricks $= \frac{n}{17}(1096.875)$

$$1850400 + \frac{n}{17}(1096.875) = (1096.875)n$$

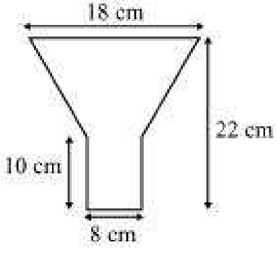
$$1850400 = \frac{16n}{17}(1096.875)$$

$n = 1792.41$

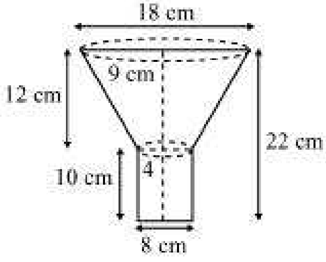
Therefore, 1792 bricks were placed in the cistern.

Question 5:

An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see the given figure).



Answer:



Radius (r_1) of upper circular end of frustum part $= \frac{18}{2} = 9 \text{ cm}$

Radius (r_2) of lower circular end of frustum part = Radius of circular end of cylindrical part

$$= \frac{8}{2} = 4 \text{ cm}$$

Height (h_1) of frustum part = 22 − 10 = 12 cm

Height (h_2) of cylindrical part = 10 cm

Slant height (l) of frustum part $= \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(9-4)^2 + (12)^2} = 13 \text{ cm}$

Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

$$= \pi(r_1 + r_2)l + 2\pi r_2 h_2$$

$$= \frac{22}{7} \times (9+4) \times 13 + 2 \times \frac{22}{7} \times 4 \times 10$$

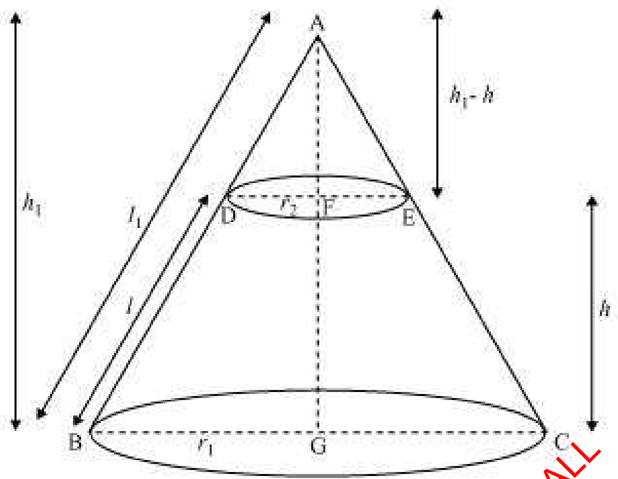
$$= \frac{22}{7}[169 + 80] = \frac{22 \times 249}{7}$$

$$= 782\frac{4}{7} \text{ cm}^2$$

Question 6:

Derive the formula for the curved surface area and total surface area of the frustum of cone.

Answer:



Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base. Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

In ΔABG and ΔADF, DF||BG

∴ ΔABG ~ ΔADF

$$\frac{DF}{BG}=\frac{AF}{AG}=\frac{AD}{AB}$$

$$\frac{r_2}{r_1}=\frac{h_1-h}{h_1}=\frac{l_1-l}{l_1}$$

$$\frac{r_2}{r_1}=1-\frac{h}{h_1}=1-\frac{l}{l_1}$$

$$1-\frac{l}{l_1}=\frac{r_2}{r_1}$$

$$\frac{l}{l_1}=1-\frac{r_2}{r_1}=\frac{r_1-r_2}{r_1}$$

$$\frac{l_1}{l}=\frac{r_1}{r_1-r_2}$$

$$l_1=\frac{r_1 l}{r_1-r_2}$$

CSA of frustum DECB = CSA of cone ABC − CSA cone ADE

$$= \pi r_1 l_1 - \pi r_2 (l_1 - l)$$

$$= \pi r_1 \left(\frac{l r_1}{r_1 - r_2} \right) - \pi r_2 \left[\frac{r_1 l}{r_1 - r_2} - l \right]$$

$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \pi r_2 \left(\frac{r_1 l - r_1 l + r_2 l}{r_1 - r_2} \right)$$

$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \frac{\pi r_2^2 l}{r_1 - r_2}$$

$$= \pi l \left[\frac{r_1^2 - r_2^2}{r_1 - r_2} \right]$$

CSA of frustum = $\pi (r_1 + r_2) l$

Total surface area of frustum = CSA of frustum + Area of upper circular end + Area of lower circular end

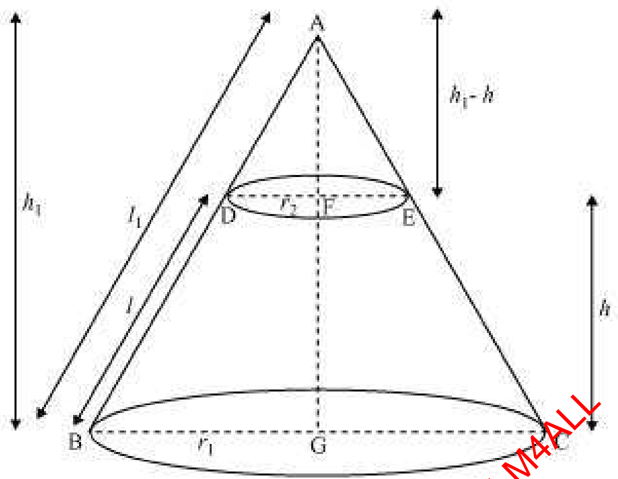
$$= \pi (r_1 + r_2) l + \pi r_2^2 + \pi r_1^2$$

$$= \pi \left[(r_1 + r_2) l + r_1^2 + r_2^2 \right]$$

Question 7:

Derive the formula for the volume of the frustum of a cone.

Answer:



Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base.

Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

In ΔABG and ΔADF, DF||BG

∴ ΔABG ~ ΔADF

$$\frac{DF}{BG}=\frac{AF}{AG}=\frac{AD}{AB}$$

$$\frac{r_2}{r_1}=\frac{h_1-h}{h_1}=\frac{l_1-l}{l_1}$$

$$\frac{r_2}{r_1}=1-\frac{h}{h_1}=1-\frac{l}{l_1}$$

$$1-\frac{h}{h_1}=\frac{r_2}{r_1}$$

$$\frac{h}{h_1}=1-\frac{r_2}{r_1}=\frac{r_1-r_2}{r_1}$$

$$\frac{h_1}{h}=\frac{r_1}{r_1-r_2}$$

$$h_1=\frac{r_1 h}{r_1-r_2}$$

Volume of frustum of cone = Volume of cone ABC – Volume of cone ADE

$$=\frac{1}{3}\pi r_1^2 h_1-\frac{1}{3}\pi r_2^2\left(h_1-h\right)$$

$$=\frac{\pi}{3}\left[r_1^2 h_1-r_2^2\left(h_1-h\right)\right]$$

$$=\frac{\pi}{3}\left[r_1^2\left(\frac{hr_1}{r_1-r_2}\right)-r_2^2\left(\frac{hr_1}{r_1-r_2}-h\right)\right]$$

$$=\frac{\pi}{3}\left[\left(\frac{hr_1^3}{r_1-r_2}\right)-r_2^2\left(\frac{hr_1-hr_1+hr_2}{r_1-r_2}\right)\right]$$

$$=\frac{\pi}{3}\left[\frac{hr_1^3}{r_1-r_2}-\frac{hr_2^3}{r_1-r_2}\right]$$

$$=\frac{\pi}{3}h\left[\frac{r_1^3-r_2^3}{r_1-r_2}\right]$$

$$=\frac{\pi}{3}h\left[\frac{\left(r_1-r_2\right)\left(r_1^2+r_2^2+r_1r_2\right)}{r_1-r_2}\right]$$

$$=\frac{1}{3}\pi h\left[r_1^2+r_2^2+r_1r_2\right]$$

Exercise 14.1

Question 1:

A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 − 2	2 − 4	4 − 6	6 − 8	8 − 10	10 − 12	12 − 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Answer:

To find the class mark (x_i) for each interval, the following relation is used.

Class mark (x_i) = $\dfrac{\text{Upper class limit} + \text{Lower class limit}}{2}$

x_i and f_ix_i can be calculated as follows.

Number of plants	**Number of houses (f_i)**	x_i	f_ix_i
0 − 2	1	1	1 × 1 = 1
2 − 4	2	3	2 × 3 = 6
4 − 6	1	5	1 × 5 = 5
6 − 8	5	7	5 × 7 = 35
8 − 10	6	9	6 × 9 = 54
10 − 12	2	11	2 ×11 = 22
12 − 14	3	13	3 × 13 = 39

Total	20		162

From the table, it can be observed that

$\sum f_i = 20$

$\sum f_i x_i = 162$

Mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

$= \dfrac{162}{20} = 8.1$

Therefore, mean number of plants per house is 8.1.

Here, direct method has been used as the values of class marks (x_i) and f_i are small.

Question 2:

Consider the following distribution of daily wages of 50 worker of a factory.

Daily wages (in Rs)	100 – 120	120 – 140	140 –1 60	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Answer:

To find the class mark for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of this data = 20

Taking 150 as assured mean (a), d_i, u_i, and f_iu_i can be calculated as follows.

Daily wages (in Rs)	Number of workers (f_i)	x_i	$d_i = x_i - 150$	$u_i = \frac{d_i}{20}$	f_iu_i
100 −120	12	110	− 40	− 2	− 24
120 − 140	14	130	− 20	− 1	− 14
140 − 160	8	150	0	0	0
160 −180	6	170	20	1	6
180 − 200	10	190	40	2	20
Total	50				− 12

From the table, it can be observed that

$$\sum f_i = 50$$
$$\sum f_iu_i = -12$$

$$\text{Mean } \bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right)h$$
$$= 150 + \left(\frac{-12}{50}\right)20$$
$$= 150 - \frac{24}{5}$$
$$= 150 - 4.8$$
$$= 145.2$$

Therefore, the mean daily wage of the workers of the factory is Rs 145.20.

Question 3:

The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs.18. Find the missing frequency *f*.

Daily pocket allowance (in Rs)	11 – 13	13 – 15	15 –17	17 – 19	19 – 21	21 – 23	23 – 25
Number of workers	7	6	9	13	*f*	5	4

Answer:

To find the class mark (x_i) for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Given that, mean pocket allowance, $\overline{x} = \text{Rs } 18$

Taking 18 as assured mean (a), d_i and $f_i d_i$ are calculated as follows.

Daily pocket allowance (in Rs)	Number of children f_i	Class mark x_i	$d_i = x_i - 18$	$f_i d_i$
11 –13	7	12	– 6	– 42
13 – 15	6	14	– 4	– 24
15 – 17	9	16	– 2	– 18
17 –19	13	18	0	0
19 – 21	*f*	20	2	2 *f*
21 – 23	5	22	4	20
23 – 25	4	24	6	24
Total	$\sum f_i = 44 + f$			$2f - 40$

From the table, we obtain

$\sum f_i = 44 + f$

$\sum f_i u_i = 2f - 40$

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$18 = 18 + \left(\frac{2f - 40}{44 + f}\right)$$

$$0 = \left(\frac{2f - 40}{44 + f}\right)$$

$2f - 40 = 0$

$2f = 40$

$f = 20$

Hence, the missing frequency, f, is 20.

Question 4:

Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows. Fine the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute	65 – 68	68 – 71	71 –74	74 – 77	77 – 80	80 – 83	83 – 86
Number of women	2	4	3	8	7	4	2

Answer:

To find the class mark of each interval (x_i), the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size, h, of this data = 3

Taking 75.5 as assumed mean (a), d_i, u_i, f_iu_i are calculated as follows.

Number of heart beats per minute	Number of women f_i	x_i	$d_i = x_i - 75.5$	$u_i = \frac{d_i}{3}$	f_iu_i
65 – 68	2	66.5	– 9	– 3	– 6
68 – 71	4	69.5	– 6	– 2	– 8
71 – 74	3	72.5	– 3	– 1	– 3
74 – 77	8	75.5	0	0	0
77 – 80	7	78.5	3	1	7
80 – 83	4	81.5	6	2	8
83 – 86	2	84.5	9	3	6
Total	30				4

From the table, we obtain

$\sum fi = 30$

$\sum f_iu_i = 4$

$$\text{Mean } \bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$$

$$= 75.5 + \left(\frac{4}{30}\right) \times 3$$

$$= 75.5 + 0.4 = 75.9$$

Therefore, mean hear beats per minute for these women are 75.9 beats per minute.

Question 5:

In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Answer:

Number of mangoes	**Number of boxes f_i**
50 – 52	15
53 – 55	110
56 – 58	135
59 – 61	115
62 – 64	25

It can be observed that class intervals are not continuous. There is a gap of 1 between two class intervals. Therefore, $\frac{1}{2}$ has to be added to the upper class limit and $\frac{1}{2}$ has to be subtracted from the lower class limit of each interval.

Class mark (x_i) can be obtained by using the following relation.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of this data = 3

Taking 57 as assumed mean (a), d_i, u_i, f_iu_i are calculated as follows.

Class interval	f_i	x_i	$d_i = x_i - 57$	$u_i = \frac{d_i}{3}$	f_iu_i
49.5 – 52.5	15	51	– 6	– 2	– 30
52.5 – 55.5	110	54	– 3	– 1	– 110
55.5 – 58.5	135	57	0	0	0
58.5 – 61.5	115	60	3	1	115
61.5 – 64.5	25	63	6	2	50
Total	400				25

It can be observed that

$\sum f_i = 400$

$\sum f_iu_i = 25$

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$$

$$= 57 + \left(\frac{25}{400}\right) \times 3$$

$$= 57 + \frac{3}{16} = 57 + 0.1875$$

$$= 57.1875$$

$$\simeq 57.19$$

Mean number of mangoes kept in a packing box is 57.19.

Step deviation method is used here as the values of f_i, d_i are big and also, there is a common multiple between all d_i.

Question 6:

The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in Rs)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Answer:

To find the class mark (x_i) for each interval, the following relation is used.

$$x_1 = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size = 50

Taking 225 as assumed mean (a), d_i, u_i, f_iu_i are calculated as follows.

Daily expenditure (in Rs)	f_i	x_i	$d_i = x_i - 225$	$u_i = \frac{d_i}{50}$	f_iu_i
100 – 150	4	125	− 100	− 2	− 8
150 – 200	5	175	− 50	− 1	− 5
200 – 250	12	225	0	0	0
250 – 300	2	275	50	1	2
300 – 350	2	325	100	2	4
Total	25				− 7

From the table, we obtain

$\sum f_i = 25$

$\sum f_i u_i = -7$

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$= 225 + \left(\frac{-7}{25}\right) \times (50)$$

$$= 225 - 14$$

$$= 211$$

Therefore, mean daily expenditure on food is Rs 211.

Question 7:

To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

concentration of SO_2 (in ppm)	**Frequency**
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4
0.20 – 0.24	2

Find the mean concentration of SO_2 in the air.

Answer:

To find the class marks for each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size of this data = 0.04

Taking 0.14 as assumed mean (a), d_i, u_i, f_iu_i are calculated as follows.

Concentration of SO_2 (in ppm)	**Frequency f_i**	**Class mark x_i**	**$d_i = x_i -$ 0.14**	$u_i = \frac{d_i}{0.04}$	**f_iu_i**
0.00 – 0.04	4	0.02	– 0.12	– 3	– 12
0.04 – 0.08	9	0.06	– 0.08	– 2	– 18
0.08 – 0.12	9	0.10	– 0.04	– 1	– 9
0.12 – 0.16	2	0.14	0	0	0
0.16 – 0.20	4	0.18	0.04	1	4
0.20 – 0.24	2	0.22	0.08	2	4
Total	30				– 31

From the table, we obtain

$$\sum f_i = 30$$

$$\sum f_iu_i = -31$$

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$$

$$= 0.14 + \left(\frac{-31}{30}\right)(0.04)$$
$$= 0.14 - 0.04133$$
$$= 0.09867$$
$$\simeq 0.099 \text{ ppm}$$

Therefore, mean concentration of SO_2 in the air is 0.099 ppm.

Question 8:

A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0 − 6	6 − 10	10 − 14	14 − 20	20 − 28	28 − 38	38 − 40
Number of students	11	10	7	4	4	3	1

Answer:

To find the class mark of each interval, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 17 as assumed mean (a), d_i and $f_i d_i$ are calculated as follows.

Number of days	**Number of students f_i**	x_i	$d_i = x_i - 17$	$f_i d_i$
0 − 6	11	3	− 14	− 154
6 − 10	10	8	− 9	− 90
10 − 14	7	12	− 5	− 35

14 – 20	4	17	0	0
20 – 28	4	24	7	28
28 – 38	3	33	16	48
38 – 40	1	39	22	22
Total	40			– 181

From the table, we obtain

$\sum f_i = 40$

$\sum f_i d_i = -181$

$$\begin{aligned} \text{Mean, } \bar{x} &= a + \left(\frac{\sum f_i d_i}{\sum f_i}\right) \\ &= 17 + \left(\frac{-181}{40}\right) \\ &= 17 - 4.525 \\ &= 12.475 \\ &\simeq 12.48 \end{aligned}$$

Therefore, the mean number of days is 12.48 days for which a student was absent.

Question 9:

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

Answer:

To find the class marks, the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) for this data = 10

Taking 70 as assumed mean (a), d_i, u_i, and f_iu_i are calculated as follows.

Literacy rate (in %)	**Number of cities f_i**	x_i	$d_i = x_i - 70$	$u_i = \frac{d_i}{10}$	f_iu_i
45 – 55	3	50	− 20	− 2	− 6
55 – 65	10	60	− 10	− 1	− 10
65 – 75	11	70	0	0	0
75 – 85	8	80	10	1	8
85 – 95	3	90	20	2	6
Total	35				− 2

From the table, we obtain

$\sum f_i = 35$

$\sum f_iu_i = -2$

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$$

$$= 70 + \left(\frac{-2}{35}\right) \times (10)$$

$$= 70 - \frac{20}{35}$$

$$= 70 - \frac{4}{7}$$

$$= 70 - 0.57$$

$$= 69.43$$

Therefore, mean literacy rate is 69.43%.

Exercise 14.2

Question 1:

The following table shows the ages of the patients admitted in a hospital during a year:

age (in years)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Answer:

To find the class marks (x_i), the following relation is used.

$$x_i = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 30 as assumed mean (a), d_i and $f_i d_i$ are calculated as follows.

Age (in years)	**Number of patients f_i**	**Class mark x_i**	**$d_i = x_i - 30$**	**$f_i d_i$**
5 – 15	6	10	– 20	– 120
15 – 25	11	20	– 10	– 110
25 – 35	21	30	0	0
35 – 45	23	40	10	230
45 – 55	14	50	20	280
55 – 65	5	60	30	150
Total	80			430

From the table, we obtain

$$\sum f_i = 80$$
$$\sum f_i d_i = 430$$
$$\text{Mean, } \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$
$$= 30 + \left(\frac{430}{80}\right)$$
$$= 30 + 5.375$$
$$= 35.375$$
$$\simeq 35.38$$

Mean of this data is 35.38. It represents that on an average, the age of a patient admitted to hospital was 35.38 years.

It can be observed that the maximum class frequency is 23 belonging to class interval 35 − 45.

Modal class = 35 − 45

Lower limit (l) of modal class = 35

Frequency (f_1) of modal class = 23

Class size (h) = 10

Frequency (f_0) of class preceding the modal class = 21

Frequency (f_2) of class succeeding the modal class = 14

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$= 35 + \left(\frac{23-21}{2(23)-21-14}\right) \times 10$$
$$= 35 + \left[\frac{2}{46-35}\right] \times 10$$
$$= 35 + \frac{20}{11}$$
$$= 35 + 1.81$$
$$= 36.8$$

Mode is 36.8. It represents that the age of maximum number of patients admitted in hospital was 36.8 years.

Question 2:

The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours)	0 − 20	20 − 40	40 − 60	60 − 80	80 − 100	100 − 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Answer:

From the data given above, it can be observed that the maximum class frequency is 61, belonging to class interval 60 − 80.

Therefore, modal class = 60 − 80

Lower class limit (l) of modal class = 60

Frequency (f_1) of modal class = 61

Frequency (f_0) of class preceding the modal class = 52

Frequency (f_2) of class succeeding the modal class = 38

Class size (h) = 20

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 60 + \left(\frac{61-52}{2(61)-52-38}\right)(20)$$

$$= 60 + \left(\frac{9}{122-90}\right)(20)$$
$$= 60 + \left(\frac{9 \times 20}{32}\right)$$
$$= 60 + \frac{90}{16} = 60 + 5.625$$
$$= 65.625$$

Therefore, modal lifetime of electrical components is 65.625 hours.

Question 3:

The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in Rs)	Number of families
1000 – 1500	24
1500 – 2000	40
2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

Answer:

It can be observed from the given data that the maximum class frequency is 40, belonging to 1500 – 2000 intervals.

Therefore, modal class = 1500 – 2000

Lower limit (l) of modal class = 1500

Frequency (f_1) of modal class = 40

Frequency (f_0) of class preceding modal class = 24

Frequency (f_2) of class succeeding modal class = 33

Class size (h) = 500

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 1500 + \left(\frac{40 - 24}{2(40) - 24 - 33}\right) \times 500$$

$$= 1500 + \left(\frac{16}{80 - 57}\right) \times 500$$

$$= 1500 + \frac{8000}{23}$$

$$= 1500 + 347.826$$

$$= 1847.826 \simeq 1847.83$$

Therefore, modal monthly expenditure was Rs 1847.83.

To find the class mark, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of the given data = 500

Taking 2750 as assumed mean (a), d_i, u_i, and $f_iu_i$are calculated as follows.

Expenditure (in Rs)	**Number of families f_i**	x_i	$d_i = x_i - 2750$	$u_i = \frac{d_i}{500}$	f_iu_i

1000 – 1500	24	1250	– 1500	– 3	– 72
1500 – 2000	40	1750	– 1000	– 2	– 80
2000 – 2500	33	2250	– 500	– 1	– 33
2500 – 3000	28	2750	0	0	0
3000 – 3500	30	3250	500	1	30
3500 – 4000	22	3750	1000	2	44
4000 – 4500	16	4250	1500	3	48
4500 – 5000	7	4750	2000	4	28
Total	200				– 35

From the table, we obtain

$$\sum f_i = 200$$

$$\sum f_i u_i = -35$$

$$\bar{x}(\text{mean}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\bar{x} = 2750 + \left(\frac{-35}{200}\right) \times 500$$

$$= 2750 - 87.5$$

$$= 2662.5$$

Therefore, mean monthly expenditure was Rs 2662.50.

Question 4:

The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	**Number of states/U.T**
15 − 20	3
20 − 25	8
25 − 30	9
30 − 35	10
35 − 40	3
40 − 45	0
45 − 50	0
50 − 55	2

Answer:

It can be observed from the given data that the maximum class frequency is 10 belonging to class interval 30 − 35.

Therefore, modal class = 30 − 35

Class size (h) = 5

Lower limit (l) of modal class = 30

Frequency (f_1) of modal class = 10

Frequency (f_0) of class preceding modal class = 9

Frequency (f_2) of class succeeding modal class = 3

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{10-9}{2(10)-9-3}\right) \times (5)$$

$$= 30 + \left(\frac{1}{20-12}\right)5$$

$$= 30 + \frac{5}{8} = 30.625$$

$$\text{Mode} = 30.6$$

It represents that most of the states/U.T have a teacher-student ratio as 30.6.

To find the class marks, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 32.5 as assumed mean (a), d_i, u_i, and f_iu_i are calculated as follows.

Number of students per teacher	**Number of states/U.T (f_i)**	x_i	$d_i = x_i - 32.5$	$u_i = \frac{d_i}{5}$	f_iu_i
15 – 20	3	17.5	– 15	– 3	– 9
20 – 25	8	22.5	– 10	– 2	– 16
25 – 30	9	27.5	– 5	– 1	– 9
30 – 35	10	32.5	0	0	0
35 – 40	3	37.5	5	1	3
40 – 45	0	42.5	10	2	0

45 – 50	0	47.5	15	3	0
50 – 55	2	52.5	20	4	8
Total	35				– 23

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$$

$$= 32.5 + \left(\frac{-23}{35}\right) \times 5$$

$$= 32.5 - \frac{23}{7} = 32.5 - 3.28$$

$$= 29.22$$

Therefore, mean of the data is 29.2.

It represents that on an average, teacher−student ratio was 29.2.

Question 5:

The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	**Number of batsmen**
3000 – 4000	4
4000 – 5000	18
5000 – 6000	9
6000 – 7000	7

7000 – 8000	6
8000 – 9000	3
9000 – 10000	1
10000 – 11000	1

Find the mode of the data.

Answer:

From the given data, it can be observed that the maximum class frequency is 18, belonging to class interval 4000 – 5000.

Therefore, modal class = 4000 – 5000

Lower limit (l) of modal class = 4000

Frequency (f_1) of modal class = 18

Frequency (f_0) of class preceding modal class = 4

Frequency (f_2) of class succeeding modal class = 9

Class size (h) = 1000

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 4000 + \left(\frac{18-4}{2(18)-4-9}\right) \times 1000$$

$$= 4000 + \left(\frac{14000}{23}\right)$$

$$= 4000 + 608.695$$

$$= 4608.695$$

Therefore, mode of the given data is 4608.7 runs

Question 6:

A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	14	13	12	20	11	15	8

Answer:

From the given data, it can be observed that the maximum class frequency is 20, belonging to 40 – 50 class intervals.

Therefore, modal class = 40 – 50

Lower limit (l) of modal class = 40

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 12

Frequency (f_2) of class succeeding modal class = 11

Class size = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left[\frac{20-12}{2(20)-12-11}\right] \times 10$$

$$= 40 + \left(\frac{80}{40-23}\right)$$

$$= 40 + \frac{80}{17}$$

$$= 40 + 4.7$$

$$= 44.7$$

Therefore, mode of this data is 44.7 cars.

Exercise 14.3

Question 1:

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	**Number of consumers**
65 − 85	4
85 − 105	5
105 − 125	13
125 − 145	20
145 − 165	14
165 − 185	8
185 − 205	4

Answer:

To find the class marks, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 135 as assumed mean (a), d_i, u_i, f_iu_i are calculated according to step deviation method as follows.

Monthly consumption (in units)	**Number of consumers (f_i)**	**x_i class mark**	**$d_i = x_i - 135$**	$u_i = \frac{d_i}{20}$	f_iu_i
65 − 85	4	75	− 60	− 3	−

					12
85 – 105	5	95	– 40	– 2	– 10
105 – 125	13	115	– 20	– 1	– 13
125 – 145	20	135	0	0	0
145 – 165	14	155	20	1	14
165 – 185	8	175	40	2	16
185 – 205	4	195	60	3	12
Total	68				7

From the table, we obtain

$\sum f_i u_i = 7$

$\sum f_i = 68$

Class size $(h) = 20$

$$\text{Mean, } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$= 135 + \frac{7}{68} \times 20$$

$$= 135 + \frac{140}{68}$$

$$= 137.058$$

From the table, it can be observed that the maximum class frequency is 20, belonging to class interval 125 – 145.

Modal class = 125 – 145

Lower limit (l) of modal class = 125

Class size (h) = 20

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 13

Frequency (f_2) of class succeeding the modal class = 14

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 125 + \left[\frac{20-13}{2(20)-13-14}\right] \times 20$$

$$= 125 + \frac{7}{13} \times 20$$

$$= 125 + \frac{140}{13} = 135.76$$

To find the median of the given data, cumulative frequency is calculated as follows.

Monthly consumption (in units)	**Number of consumers**	**Cumulative frequency**
65 – 85	4	4
85 – 105	5	4 + 5 = 9
105 – 125	13	9 + 13 = 22
125 – 145	20	22 + 20 = 42
145 – 165	14	42 + 14 = 56
165 – 185	8	56 + 8 = 64
185 – 205	4	64 + 4 = 68

From the table, we obtain

$n = 68$

Cumulative frequency (*cf*) just greater than $\frac{n}{2}\left(\text{i.e., } \frac{68}{2}=34\right)$ is 42, belonging to interval 125 − 145.

Therefore, median class = 125 − 145

Lower limit (*l*) of median class = 125

Class size (*h*) = 20

Frequency (*f*) of median class = 20

Cumulative frequency (*cf*) of class preceding median class = 22

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 125 + \left(\frac{34-22}{20}\right) \times 20$$

$$= 125 + 12$$

$$= 137$$

Therefore, median, mode, mean of the given data is 137, 135.76, and 137.05 respectively.

The three measures are approximately the same in this case.

Question 2:

If the median of the distribution is given below is 28.5, find the values of *x* and *y*.

Class interval	Frequency
0 − 10	5
10 − 20	x
20 − 30	20

30 – 40	15
40 – 50	y
50 – 60	5
Total	60

Answer:

The cumulative frequency for the given data is calculated as follows.

Class interval	**Frequency**	**Cumulative frequency**
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$
Total (n)	60	

From the table, it can be observed that $n = 60$

$45 + x + y = 60$

$x + y = 15$ (1)

Median of the data is given as 28.5 which lies in interval 20 – 30.

Therefore, median class = 20 – 30

Lower limit (l) of median class = 20

Cumulative frequency (cf) of class preceding the median class = $5 + x$

Frequency (f) of median class = 20

Class size (h) = 10

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$28.5 = 20 + \left[\frac{\frac{60}{2} - (5 + x)}{20}\right] \times 10$$

$$8.5 = \left(\frac{25 - x}{2}\right)$$

$$17 = 25 - x$$

$$x = 8$$

From equation (1),

$8 + y = 15$

$y = 7$

Hence, the values of x and y are 8 and 7 respectively

Question 3:

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45

Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Answer:

Here, class width is not the same. There is no requirement of adjusting the frequencies according to class intervals. The given frequency table is of less than type represented with upper class limits. The policies were given only to persons with age 18 years onwards but less than 60 years. Therefore, class intervals with their respective cumulative frequency can be defined as below.

Age (in years)	**Number of policy holders (f_i)**	**Cumulative frequency (cf)**
18 − 20	2	2
20 − 25	6 − 2 = 4	6
25 − 30	24 − 6 = 18	24
30 − 35	45 − 24 = 21	45
35 − 40	78 − 45 = 33	78
40 − 45	89 − 78 = 11	89
45 − 50	92 − 89 = 3	92
50 − 55	98 − 92 = 6	98

55 − 60	100 − 98 = 2	100
Total (n)		

From the table, it can be observed that n = 100.

Cumulative frequency (cf) just greater than $\frac{n}{2}\left(\text{i.e., } \frac{100}{2} = 50\right)$ is 78, belonging to interval 35 − 40.

Therefore, median class = 35 − 40

Lower limit (l) of median class = 35

Class size (h) = 5

Frequency (f) of median class = 33

Cumulative frequency (cf) of class preceding median class = 45

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 35 + \left(\frac{50 - 45}{33}\right) \times 5$$

$$= 35 + \frac{25}{33}$$

$$= 35.76$$

Therefore, median age is 35.76 years.

Question 4:

The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	**Number or leaves f_i**
118 − 126	3

127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the median length of the leaves.

(**Hint:** The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 – 126.5, 126.5 – 135.5… 171.5 – 180.5)

Answer:

The given data does not have continuous class intervals. It can be observed that the difference between two class intervals is 1. Therefore, $\frac{1}{2} = 0.5$ has to be added and subtracted to upper class limits and lower class limits respectively.

Continuous class intervals with respective cumulative frequencies can be represented as follows.

Length (in mm)	**Number or leaves f_i**	**Cumulative frequency**
117.5 – 126.5	3	3
126.5 – 135.5	5	3 + 5 = 8
135.5 – 144.5	9	8 + 9 = 17
144.5 – 153.5	12	17 + 12 = 29

153.5 − 162.5	5	29 + 5 = 34
162.5 − 171.5	4	34 + 4 = 38
171.5 − 180.5	2	38 + 2 = 40

From the table, it can be observed that the cumulative frequency just greater than $\frac{n}{2}\left(\text{i.e., } \frac{40}{2} = 20\right)$ is 29, belonging to class interval 144.5 − 153.5.

Median class = 144.5 − 153.5

Lower limit (l) of median class = 144.5

Class size (h) = 9

Frequency (f) of median class = 12

Cumulative frequency (cf) of class preceding median class = 17

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$$= 144.5 + \left(\frac{20 - 17}{12}\right) \times 9$$

$$= 144.5 + \frac{9}{4} = 146.75$$

Therefore, median length of leaves is 146.75 mm.

Question 5:

Find the following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	**Number of lamps**
1500 − 2000	14

2000 – 2500	56
2500 – 3000	60
3000 – 3500	86
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

Find the median life time of a lamp.

Answer:

Thecumulative frequencies with their respective class intervals are as follows.

Life time	**Number of lamps (f_i)**	**Cumulative frequency**
1500 – 2000	14	14
2000 – 2500	56	14 + 56 = 70
2500 – 3000	60	70 + 60 = 130
3000 – 3500	86	130 + 86 = 216
3500 – 4000	74	216 + 74 = 290
4000 – 4500	62	290 + 62 = 352
4500 – 5000	48	352 + 48 = 400
Total (n)	400	

It can be observed that the cumulative frequency just greater than

$\frac{n}{2}\left(\text{i.e., } \frac{400}{2}=200\right)$ is 216, belonging to class interval 3000 − 3500.

Median class = 3000 − 3500

Lower limit (l) of median class = 3000

Frequency (f) of median class = 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$$= 3000 + \left(\frac{200-130}{86}\right) \times 500$$

$$= 3000 + \frac{70 \times 500}{86}$$

= 3406.976

Therefore, median life time of lamps is 3406.98 hours.

Question 6:

100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 − 4	4 − 7	7 − 10	10 − 13	13 − 16	16 − 19
Number of surnames	6	30	40	6	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Answer:

The cumulative frequencies with their respective class intervals are as follows.

Number of letters	**Frequency (f_i)**	**Cumulative frequency**
1 – 4	6	6
4 – 7	30	30 + 6 = 36
7 – 10	40	36 + 40 = 76
10 – 13	16	76 + 16 = 92
13 – 16	4	92 + 4 = 96
16 – 19	4	96 + 4 = 100
Total (n)	100	

It can be observed that the cumulative frequency just greater than $\frac{n}{2}\left(\text{i.e., } \frac{100}{2} = 50\right)$ is 76, belonging to class interval 7 – 10.

Median class = 7 – 10

Lower limit (l) of median class = 7

Cumulative frequency (cf) of class preceding median class = 36

Frequency (f) of median class = 40

Class size (h) = 3

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$= 7 + \left(\frac{50-36}{40}\right) \times 3$

$= 7 + \frac{14 \times 3}{40}$

= 8.05

To find the class marks of the given class intervals, the following relation is used.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Taking 11.5 as assumed mean (a), d_i, u_i, and f_iu_i are calculated according to step deviation method as follows.

Number of letters	**Number of surnames f_i**	x_i	$d_i = x_i - 11.5$	$u_i = \frac{d_i}{3}$	f_iu_i
1 – 4	6	2.5	– 9	– 3	– 18
4 – 7	30	5.5	– 6	– 2	– 60
7 – 10	40	8.5	– 3	– 1	– 40
10 – 13	16	11.5	0	0	0
13 – 16	4	14.5	3	1	4
16 – 19	4	17.5	6	2	8
Total	100				– 106

From the table, we obtain

$\Sigma f_iu_i = -106$

$\Sigma f_i = 100$

Mean, $\overline{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$

$= 11.5 + \left(\frac{-106}{100}\right) \times 3$

= 11.5 – 3.18 = 8.32

The data in the given table can be written as

Number of letters	**Frequency (f_i)**
1 – 4	6
4 – 7	30
7 – 10	40
10 – 13	16
13 – 16	4
16 – 19	4
Total (n)	100

From the table, it can be observed that the maximum class frequency is 40 belonging to class interval 7 – 10.

Modal class = 7 – 10

Lower limit (l) of modal class = 7

Class size (h) = 3

Frequency (f_1) of modal class = 40

Frequency (f_0) of class preceding the modal class = 30

Frequency (f_2) of class succeeding the modal class = 16

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 7 + \left[\frac{40-30}{2(40)-30-16}\right] \times 3$$

$$= 7 + \frac{10}{34} \times 3$$

$$= 7 + \frac{30}{34} = 7.88$$

Therefore, median number and mean number of letters in surnames is 8.05 and 8.32 respectively while modal size of surnames is 7.88.

Question 7:

The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
Number of students	2	3	8	6	6	3	2

Answer:

The cumulative frequencies with their respective class intervals are as follows.

Weight (in kg)	**Frequency (*fi*)**	**Cumulative frequency**
40 – 45	2	2
45 – 50	3	2 + 3 = 5
50 – 55	8	5 + 8 = 13
55 – 60	6	13 + 6 = 19

60 – 65	6	19 + 6 = 25
65 – 70	3	25 + 3 = 28
70 – 75	2	28 + 2 = 30
Total (n)	30	

Cumulative frequency just greater than $\frac{n}{2}\left(\text{i.e., } \frac{30}{2} = 15\right)$ is 19, belonging to class interval 55 – 60.

Median class = 55 – 60

Lower limit (l) of median class = 55

Frequency (f) of median class = 6

Cumulative frequency (cf) of median class = 13

Class size (h) = 5

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$$= 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

$$= 55 + \frac{10}{6}$$

= 56.67

Therefore, median weight is 56.67 kg.

Exercise 14.4

Question 1:

The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

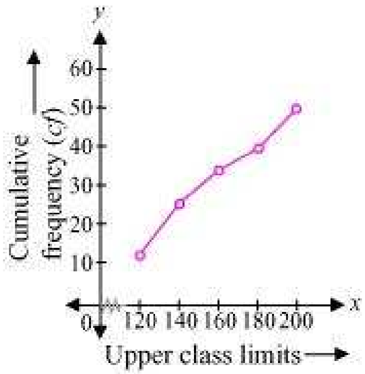
Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Answer:

The frequency distribution table of less than type is as follows.

Daily income (in Rs) (upper class limits)	**Cumulative frequency**
Less than 120	12
Less than 140	12 + 14 = 26
Less than 160	26 + 8 = 34
Less than 180	34 + 6 = 40
Less than 200	40 + 10 = 50

Taking upper class limits of class intervals on x-axis and their respective frequencies on y-axis, its ogive can be drawn as follows.



Question 2:

During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

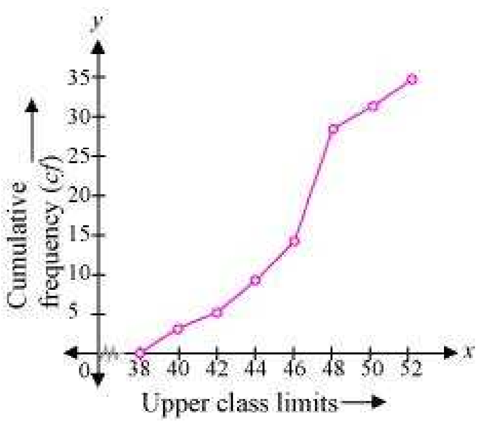
Draw a less than type ogive for the given data. Hence obtain the median weight from the graph verify the result by using the formula.

Answer:

The given cumulative frequency distributions of less than type are

Weight (in kg) upper class limits	**Number of students (cumulative frequency)**
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

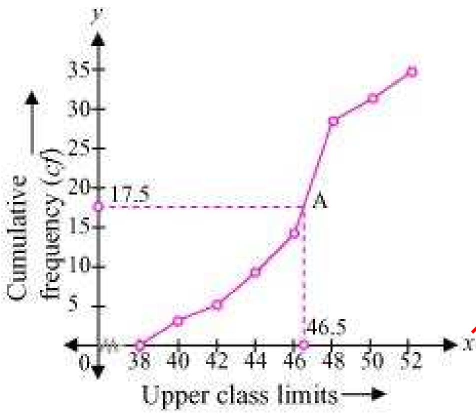
Taking upper class limits on *x*-axis and their respective cumulative frequencies on *y*-axis, its ogive can be drawn as follows.



Here, $n = 35$

So, $\frac{n}{2} = 17.5$

Mark the point A whose ordinate is 17.5 and its x-coordinate is 46.5. Therefore, median of this data is 46.5.



It can be observed that the difference between two consecutive upper class limits is 2. The class marks with their respective frequencies are obtained as below.

Weight (in kg)	**Frequency (*f*)**	**Cumulative frequency**
Less than 38	0	0
38 − 40	3 − 0 = 3	3
40 − 42	5 − 3 = 2	5
42 − 44	9 − 5 = 4	9
44 − 46	14 − 9 = 5	14
46 − 48	28 − 14 = 14	28
48 − 50	32 − 28 = 4	32

50 – 52	35 – 32 = 3	35
Total (n)	35	

The cumulative frequency just greater than $\frac{n}{2}\left(\text{i.e., } \frac{35}{2} = 17.5\right)$ is 28, belonging to class interval 46 – 48.

Median class = 46 – 48

Lower class limit (l) of median class = 46

Frequency (f) of median class = 14

Cumulative frequency (cf) of class preceding median class = 14

Class size (h) = 2

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$$

$$= 46 + \frac{3.5}{7}$$

$$= 46.5$$

Therefore, median of this data is 46.5.

Hence, the value of median is verified.

Question 3:

The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80

Number of farms	2	8	12	24	38	16

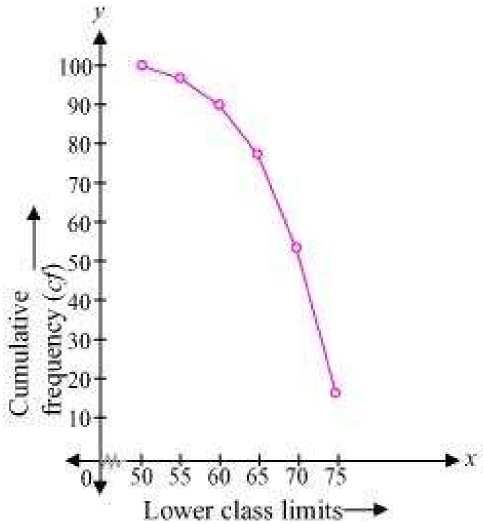
Change the distribution to a more than type distribution and draw ogive.

Answer:

The cumulative frequency distribution of more than type can be obtained as follows.

Production yield (lower class limits)	**Cumulative frequency**
more than or equal to 50	100
more than or equal to 55	100 − 2 = 98
more than or equal to 60	98 − 8 = 90
more than or equal to 65	90 − 12 = 78
more than or equal to 70	78 − 24 = 54
more than or equal to 75	54 − 38 = 16

Taking the lower class limits on x-axis and their respective cumulative frequencies on y-axis, its ogive can be obtained as follows.



y
100
90
80
70
60
50
40
30
20
10
0
Cumulative frequency (cf)
50 55 60 65 70 75
x
Lower class limits

Exercise 15.1

Question 1:

Complete the following statements:

(i) Probability of an event E + Probability of the event 'not E' = ________.

(ii) The probability of an event that cannot happen is __________. Such as event is called __________.

(iii) The probability of an event that is certain to happen is __________. Such as event is called _________.

(iv) The sum of the probabilities of all the elementary events of an experiment is __________.

(v) The probability of an event is greater than or equal to ________ and less than or equal to ________.

Answer:

(i) 1

(ii) 0, impossible event

(iii) 1, sure event or certain event

(iv) 1

(v) 0, 1

Question 2:

Which of the following experiments have equally likely outcomes? Explain.

(i) A driver attempts to start a car. The car starts or does not start.

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

(iii) A trial is made to answer a true-false question. The answer is right or wrong.

(iv) A baby is born. It is a boy or a girl.

Answer:

(i) It is not an equally likely event, as it depends on various factors such as whether the car will start or not. And factors for both the conditions are not the same.

(ii) It is not an equally likely event, as it depends on the player's ability and there is no information given about that.

(iii) It is an equally likely event.

(iv) It is an equally likely event.

Question 3:

Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

Answer:

When we toss a coin, the possible outcomes are only two, head or tail, which are equally likely outcomes. Therefore, the result of an individual toss is completely unpredictable.

Question 4:

Which of the following cannot be the probability of an event?

(A) $\frac{2}{3}$ (B) -1.5 (C) 15% (D) 0.7

Answer:

Probability of an event (E) is always greater than or equal to 0. Also, it is always less than or equal to one. This implies that the probability of an event cannot be negative or greater than 1. Therefore, out of these alternatives, −1.5 cannot be a probability of an event.

Hence, (B)

Question 5:

If P(E) = 0.05, what is the probability of 'not E'?

Answer:

We know that,

$$P(\overline{E}) = 1 - P(E)$$

$$P(\overline{E}) = 1 - 0.05$$

$$= 0.95$$

Therefore, the probability of 'not E' is 0.95.

Question 6:

A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Answer:

(i) The bag contains lemon flavoured candies only. It does not contain any orange flavoured candies. This implies that every time, she will take out only lemon flavoured candies. Therefore, event that Malini will take out an orange flavoured candy is an impossible event.

Hence, P (an orange flavoured candy) = 0

(ii)As the bag has lemon flavoured candies, Malini will take out only lemon flavoured candies. Therefore, event that Malini will take out a lemon flavoured candy is a sure event.

P (a lemon flavoured candy) = 1

Question 7:

It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Answer:

Probability that two students are not having same birthday P ($\overline{E}$) = 0.992

Probability that two students are having same birthday P (E) = 1 − P ($\overline{E}$)

= 1 − 0.992

= 0.008

Question 8:

A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?

Answer:

(i) Total number of balls in the bag = 8

$$\text{Probability of getting a red ball} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{3}{8}$$

(ii) Probability of not getting red ball

= 1 − Probability of getting a red ball

$$= 1 - \frac{3}{8}$$

$$= \frac{5}{8}$$

Question 9:

A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white? (iii) not green?

Answer:

Total number of marbles = 5 + 8 + 4

= 17

(i)Number of red marbles = 5

$$\text{Probability of getting a red marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{5}{17}$$

(ii)Number of white marbles = 8

$$\text{Probability of getting a white marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{8}{17}$$

(iii)Number of green marbles = 4

$$\text{Probability of getting a green marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{4}{17}$$

Probability of not getting a green marble $= 1 - \frac{4}{17} = \frac{13}{17}$

Question 10:

A piggy bank contains hundred 50 p coins, fifty Rs 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) Will be a 50 p coin?

(ii) Will not be a Rs.5 coin?

Answer:

Total number of coins in a piggy bank = 100 + 50 + 20 + 10

= 180

(i) Number of 50 p coins = 100

$$\text{Probability of getting a 50 p coin} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{100}{180} = \frac{5}{9}$$

(ii) Number of Rs 5 coins = 10

$$\text{Probability of getting a Rs 5 coin} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{10}{180} = \frac{1}{18}$$

Probability of not getting a Rs 5 coin $= 1 - \frac{1}{18}$

$$= \frac{17}{18}$$

Question 11:

Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see the given figure). What is the probability that the fish taken out is a male fish?

Answer:

Total number of fishes in a tank

= Number of male fishes + Number of female fishes

= 5 + 8 = 13

$$\text{Probability of getting a male fish} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{5}{13}$$

Question 12:

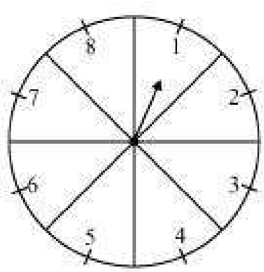
A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see the given figure), and these are equally likely outcomes. What is the probability that it will point at

(i) 8?

(ii) an odd number?

(iii) a number greater than 2?

(iv) a number less than 9?



Answer:

Total number of possible outcomes = 8

(i) $\text{Probability of getting 8} = \dfrac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} = \dfrac{1}{8}$

(ii) Total number of odd numbers on spinner = 4

$$\text{Probability of getting an odd number} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$
$$= \frac{4}{8} = \frac{1}{2}$$

(iii) The numbers greater than 2 are 3, 4, 5, 6, 7, and 8.

Therefore, total numbers greater than 2 = 6

Probability of getting a number greater than 2

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} = \frac{6}{8} = \frac{3}{4}$$

(iv) The numbers less than 9 are 1, 2, 3, 4, 6, 7, and 8.

Therefore, total numbers less than 9 = 8

Probability of getting a number less than 9 $= \dfrac{8}{8} = 1$

Question 13:

A die is thrown once. Find the probability of getting

(i) a prime number;

(ii) a number lying between 2 and 6;

(iii) an odd number.

Answer:

The possible outcomes when a dice is thrown = {1, 2, 3, 4, 5, 6}

Number of possible outcomes of a dice = 6

(i) Prime numbers on a dice are 2, 3, and 5.

Total prime numbers on a dice = 3

Probability of getting a prime number = $\frac{3}{6}=\frac{1}{2}$

(ii) Numbers lying between 2 and 6 = 3, 4, 5

Total numbers lying between 2 and 6 = 3

Probability of getting a number lying between 2 and 6 $=\frac{3}{6}=\frac{1}{2}$

(iii) Odd numbers on a dice = 1, 3, and 5

Total odd numbers on a dice = 3

Probability of getting an odd number $=\frac{3}{6}=\frac{1}{2}$

Question 14:

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour

(ii) a face card

(iii) a red face card

(iv) the jack of hearts

(v) a spade

(vi) the queen of diamonds

Answer:

Total number of cards in a well-shuffled deck = 52

(i) Total number of kings of red colour = 2

P (getting a king of red colour) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{2}{52} = \frac{1}{26}$

(ii) Total number of face cards = 12

P (getting a face card) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{12}{52} = \frac{3}{13}$

(iii) Total number of red face cards = 6

P (getting a red face card) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{6}{52} = \frac{3}{26}$

(iv) Total number of Jack of hearts = 1

P (getting a Jack of hearts) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{1}{52}$

(v) Total number of spade cards = 13

P (getting a spade card) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{13}{52} = \frac{1}{4}$

(vi) Total number of queen of diamonds = 1

P (getting a queen of diamond) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{1}{52}$

Question 15:

Five cards−−the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Answer:

(i) Total number of cards = 5

Total number of queens = 1

P (getting a queen) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{1}{5}$

(ii) When the queen is drawn and put aside, the total number of remaining cards will be 4.

(a) Total number of aces = 1

P (getting an ace) $= \frac{1}{4}$

(b) As queen is already drawn, therefore, the number of queens will be 0.

P (getting a queen) $= \frac{0}{4} = 0$

Question 16:

12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Answer:

Total number of pens = 12 + 132 = 144

Total number of good pens = 132

P (getting a good pen) $= \dfrac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \dfrac{132}{144} = \dfrac{11}{12}$

Question 17:

(i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Answer:

(i) Total number of bulbs = 20

Total number of defective bulbs = 4

P (getting a defective bulb) $= \dfrac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \dfrac{4}{20} = \dfrac{1}{5}$

(ii) Remaining total number of bulbs = 19

Remaining total number of non-defective bulbs = 16 − 1 = 15

P (getting a not defective bulb) $= \dfrac{15}{19}$

Question 18:

A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two-digit number

(ii) a perfect square number

(iii) a number divisible by 5.

Answer:

Total number of discs = 90

(i) Total number of two-digit numbers between 1 and 90 = 81

P (getting a two-digit number) $= \frac{81}{90} = \frac{9}{10}$

(ii) Perfect squares between 1 and 90 are 1, 4, 9, 16, 25, 36, 49, 64, and 81. Therefore, total number of perfect squares between 1 and 90 is 9.

P (getting a perfect square) $= \frac{9}{90} = \frac{1}{10}$

(iii) Numbers that are between 1 and 90 and divisible by 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, and 90. Therefore, total numbers divisible by 5 = 18

Probability of getting a number divisible by 5 $= \frac{18}{90} = \frac{1}{5}$

Question 19:

A child has a die whose six faces shows the letters as given below:

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A? (ii) D?

Answer:

Total number of possible outcomes on the dice = 6

(i) Total number of faces having A on it = 2

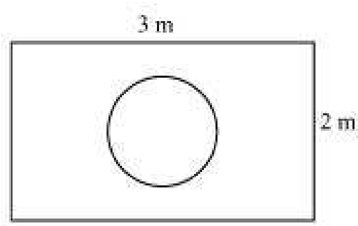
P (getting A) $= \frac{2}{6} = \frac{1}{3}$

(ii) Total number of faces having D on it = 1

P (getting D) $= \frac{1}{6}$

Question 20:

Suppose you drop a die at random on the rectangular region shown in the given figure. What is the probability that it will land inside the circle with diameter 1 m?



Answer:

Area of rectangle = $l \times b = 3 \times 2 = 6\text{ m}^2$

Area of circle (of diameter 1 m) $= \pi r^2 = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}\text{ m}^2$

P (die will land inside the circle) $= \frac{\frac{\pi}{4}}{6} = \frac{\pi}{24}$

Question 21:

A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it?

(ii) She will not buy it?

Answer:

Total number of pens = 144

Total number of defective pens = 20

Total number of good pens = 144 − 20 = 124

(i) Probability of getting a good pen $= \frac{124}{144} = \frac{31}{36}$

P (Nuri buys a pen) $= \frac{31}{36}$

(ii) P (Nuri will not buy a pen) $= 1 - \frac{31}{36} = \frac{5}{36}$

Question 22:

Two dice, one blue and one grey, are thrown at the same time.

(i) Write down all the possible outcomes and complete the following table:

Event: Sum of two dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument?

Answer:

(i) It can be observed that,

To get the sum as 2, possible outcomes = (1, 1)

To get the sum as 3, possible outcomes = (2, 1) and (1, 2)

To get the sum as 4, possible outcomes = (3, 1), (1, 3), (2, 2)

To get the sum as 5, possible outcomes = (4, 1), (1, 4), (2, 3), (3, 2)

To get the sum as 6, possible outcomes = (5, 1), (1, 5), (2, 4), (4, 2), (3, 3)

To get the sum as 7, possible outcomes = (6, 1), (1, 6), (2, 5), (5, 2), (3, 4), (4, 3)

To get the sum as 8, possible outcomes = (6, 2), (2, 6), (3, 5), (5, 3), (4, 4)

To get the sum as 9, possible outcomes = (3, 6), (6, 3), (4, 5), (5, 4)

To get the sum as 10, possible outcomes = (4, 6), (6, 4), (5, 5)

To get the sum as 11, possible outcomes = (5, 6), (6, 5)

To get the sum as 12, possible outcomes = (6, 6)

Event: Sum of two dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii)Probability of each of these sums will not be $\frac{1}{11}$ as these sums are not equally likely

Question 23:

A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Answer:

The possible outcomes are

{HHH, TTT, HHT, HTH, THH, TTH, THT, HTT}

Number of total possible outcomes = 8

Number of favourable outcomes = 2 {i.e., TTT and HHH}

P (Hanif will win the game) $= \frac{2}{8} = \frac{1}{4}$

P (Hanif will lose the game) $=1-\frac{1}{4}=\frac{3}{4}$

Question 24:

A die is thrown twice. What is the probability that

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

[**Hint:** Throwinga die twice and throwing two dice simultaneously are treated as the same experiment].

Answer:

Total number of outcomes = 6 × 6

= 36

(i)Total number of outcomes when 5 comes up on either time are (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5)

Hence, total number of favourable cases = 11

P (5 will come up either time) $=\frac{11}{36}$

P (5 will not come up either time) $=1-\frac{11}{36}=\frac{25}{36}$

(ii)Total number of cases, when 5 can come at least once = 11

P (5 will come at least once) $=\frac{11}{36}$

Question 25:

Which of the following arguments are correct and which are not correct? Give reasons for your answer.

(i) If two coins are tossed simultaneously there are three possible outcomes−−two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

(ii) If a die is thrown, there are two possible outcomes−−an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Answer:

(i) Incorrect

When two coins are tossed, the possible outcomes are (H, H), (H, T), (T, H), and (T, T). It can be observed that there can be one of each in two possible ways − (H, T), (T, H).

Therefore, the probability of getting two heads is $\frac{1}{4}$, the probability of getting two tails is $\frac{1}{4}$, and the probability of getting one of each is $\frac{1}{2}$.

It can be observed that for each outcome, the probability is not $\frac{1}{3}$.

(ii) Correct

When a dice is thrown, the possible outcomes are 1, 2, 3, 4, 5, and 6. Out of these, 1, 3, 5 are odd and 2, 4, 6 are even numbers.

Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Exercise 15.2

Question 1:

Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on

(i) the same day? (ii) consecutive days? (iii) different days?

Answer:

There are a total of 5 days. Shyam can go to the shop in 5 ways and Ekta can go to the shop in 5 ways.

Therefore, total number of outcomes = 5 × 5 = 25

(i) They can reach on the same day in 5 ways.

i.e., (t, t), (w, w), (th, th), (f, f), (s, s)

P (both will reach on same day) $= \frac{5}{25} = \frac{1}{5}$

(ii) They can reach on consecutive days in these 8 ways - (t, w), (w, th), (th, f), (f, s), (w, t), (th, w), (f, th), (s, f).

Therefore, P (both will reach on consecutive days) $= \frac{8}{25}$

(iii) P (both will reach on same day) $= \frac{1}{5}$ [(From (i)]

P (both will reach on different days) $= 1 - \frac{1}{5} = \frac{4}{5}$

Question 2:

A die is numbered in such a way that its faces show the number 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:

Number in first throw

Number in second throw

+	1	2	2	3	3	6
1	1	2	2	3	3	6
2	3	4	4	5	5	8
2					5	
3						
3			5			9
6	7	8	8	9	9	12

What is the probability that the total score is

(i) even? (ii) 6? (iii) at least 6?

Answer:

+	1 2 2 3 3 6
1	2 3 3 4 4 7
2	3 4 4 5 5 8
2	3 4 4 5 5 8
3	4 5 5 6 6 9
3	4 5 5 6 6 9
6	7 8 8 9 9 12

Total number of possible outcomes when two dice are thrown = 6 × 6 = 36

(i) Total times when the sum is even = 18

P (getting an even number) $= \frac{18}{36} = \frac{1}{2}$

(ii) Total times when the sum is 6 = 4

P (getting sum as 6) $= \frac{4}{36} = \frac{1}{9}$

P (getting sum at least 6) $= \frac{15}{36} = \frac{5}{12}$

Question 3:

A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is *double* that of a red ball, determine the number of blue balls in the bag.

Answer:

Let the number of blue balls be *x*.

Number of red balls = 5

Total number of balls = *x* + 5

P (getting a red ball) $= \frac{5}{x+5}$

P (getting a blue ball) $= \frac{x}{x+5}$

Given that,

$$2\left(\frac{5}{x+5}\right) = \frac{x}{x+5}$$

$$10(x+5) = x^2 + 5x$$

$$x^2 - 5x - 50 = 0$$

$$x^2 - 10x + 5x - 50 = 0$$

$$x(x-10) + 5(x-10) = 0$$

$$(x-10)(x+5) = 0$$

$$\text{Either } x - 10 = 0 \text{ or } x + 5 = 0$$

$$x = 10 \text{ or } x = -5$$

However, the number of balls cannot be negative.

Hence, number of blue balls = 10

Question 4:

A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

Answer:

Total number of balls = 12

Total number of black balls = x

P (getting a black ball) = $\frac{x}{12}$

If 6 more black balls are put in the box, then

Total number of balls = 12 + 6 = 18

Total number of black balls = x + 6

P (getting a black ball now) $= \frac{x+6}{18}$

According to the condition given in the question,

$$2\left(\frac{x}{12}\right)=\frac{x+6}{18}$$

$$3x = x + 6$$

$$2x = 6$$

$$x = 3$$

Question 5:

A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue balls in the jar.

Answer:

Total number of marbles = 24

Let the total number of green marbles be x.

Then, total number of blue marbles = 24 − x

P (getting a given marble) $= \frac{x}{24}$

According to the condition given in the question,

$$\frac{x}{24} = \frac{2}{3}$$

$$x = 16$$

Therefore, total number of green marbles in the jar = 16

Hence, total number of blue marbles = 24 − x = 24 − 16 = 8